Universidade Federal de Rio de Janeiro
Mathematical Institute
10/10/2022

Chasing patterns: diffusion-driven instabilities in networked and high-order systems

Riccardo Muolo, Namur Institute for Complex Systems (naXys),
Department of Mathematics, Université de Namur (Belgium)

funded by

My background

BSc Physics and MSc Applied Mathematics
Firenze

PhD Systems Biology (1 year - quit)
Amsterdam

Teaching assistant Mathematics
Namur

Since Jan 2020
PhD Applied Mathematics (exp. 2023)
Namur

Université de Namur - naXys

Prof. Teo Carletti

Dynamics on networks and beyond group

pattern-formation, random walks,
synchronization and more

group web page

naxys
Namur Institute for Complex Systems

Instituto de Matemática
Universidade Federal do Rio de Janeiro

Erasmus+

Possibility of joint Master Thesis and co-supervised PhDs

Outiline

Dynamical systems on networks

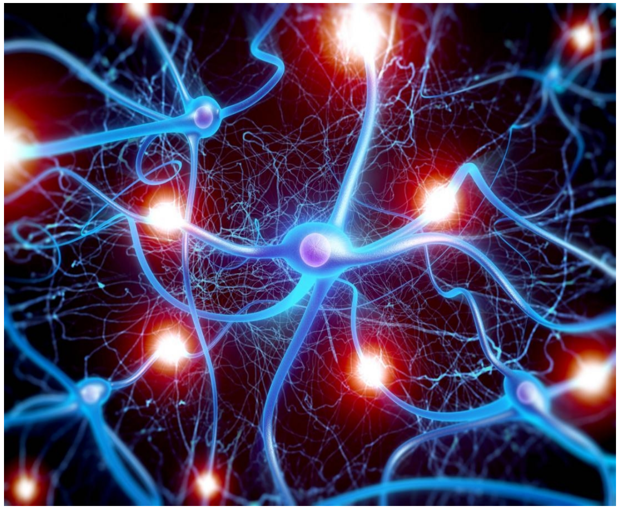
Turing theory of pattern formation

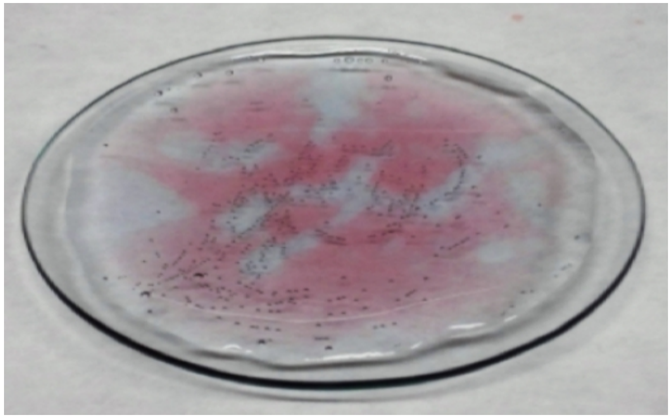
Turing theory on networks

Effects of non-normality

High-order interactions

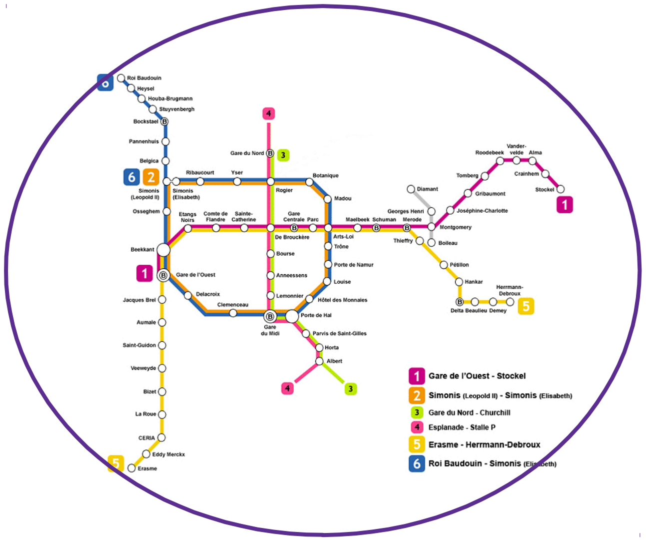
Patterns in nature

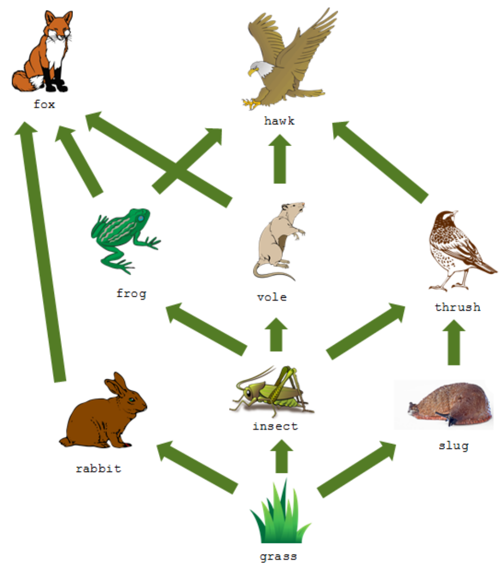




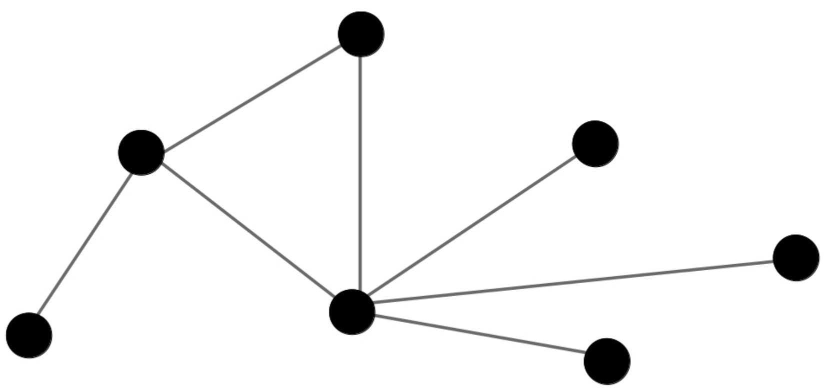
naxys
Namur Institute for Complex Systems

Networks





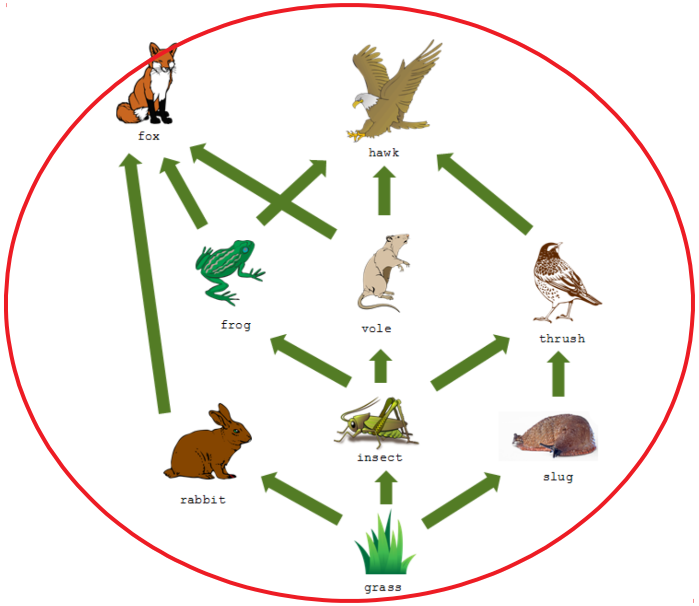
symmetric (undirected)



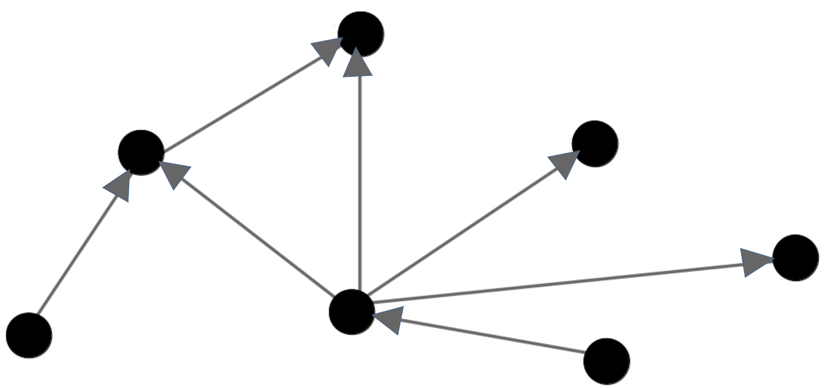
$$A_{ij} = \begin{cases} w_{ij} \in \mathbb{R}^+ \\ 0 \end{cases}$$

if there is a link between nodes i and j

Networks



asymmetric (directed)



$$A_{ij} = \begin{cases} w_{ij} \in \mathbb{R}^+ \\ 0 \end{cases}$$

if there is a directed link
from node j
to node i

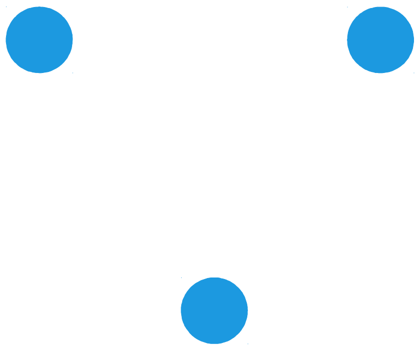
Coupled oscillators on networks

$$\dot{\vec{x}}_i = \vec{f}(\vec{x}_i)$$

Coupled oscillators on networks

$$\dot{\vec{x}}_i = \vec{f}(\vec{x}_i)$$

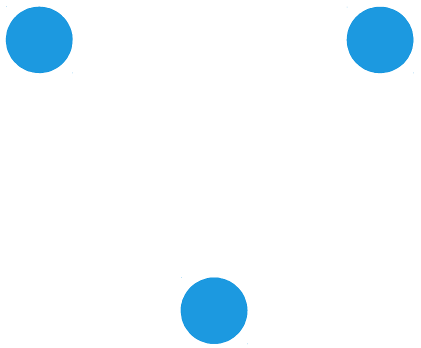
dynamics of x_i



Coupled oscillators on networks

$$\dot{\vec{x}}_i = \vec{f}(\vec{x}_i) + \sigma_1 \sum_j A_{ij}^{(1)} \vec{g}^{(1)}(\vec{x}_i, \vec{x}_j)$$

dynamics of x_i



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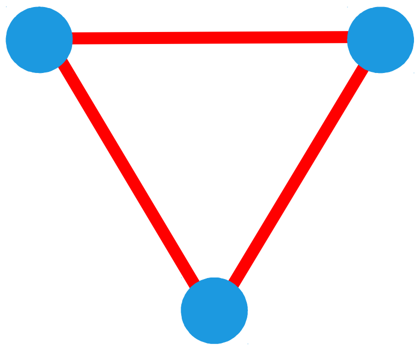
pairwise coupling



Master Stability Function (MSF)

VOLUME 80, NUMBER 10 PHYSICAL REVIEW LETTERS 9 MARCH 1998

Master Stability Functions for Synchronized Coupled Systems

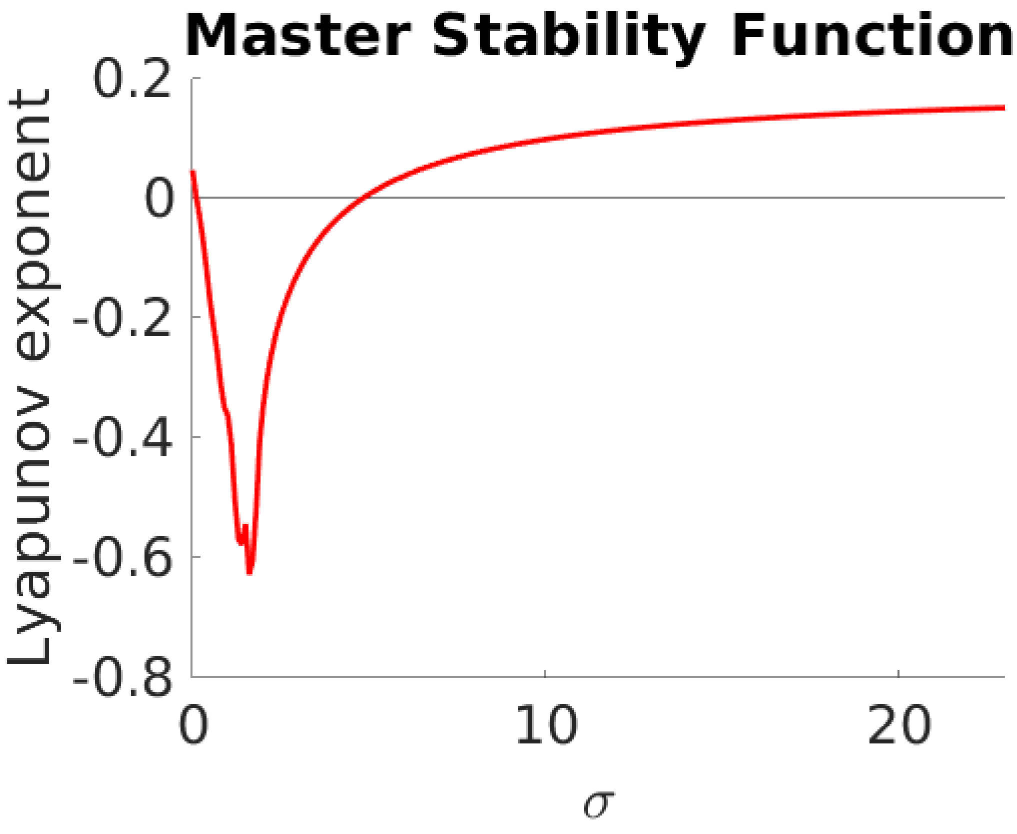
Louis M. Pecora and Thomas L. Carroll
Code 6343, Naval Research Laboratory, Washington, D.C. 20375
(Received 7 July 1997)

$$\dot{\xi} = [\mathbf{1}_N \otimes D\mathbf{F} + \sigma \mathbf{G} \otimes D\mathbf{H}]\xi$$

$$\dot{\xi}_k = [D\mathbf{F} + \sigma \gamma_k D\mathbf{H}]\xi_k$$

Rössler

$$\begin{cases} \dot{x}_i = -y_i - z_i + \sigma \sum_{j=1}^{N} L_{ij} x_j \\ \dot{y}_i = x_i + a y_i \\ \dot{z}_i = b + z_i(x_i - c) \end{cases}$$



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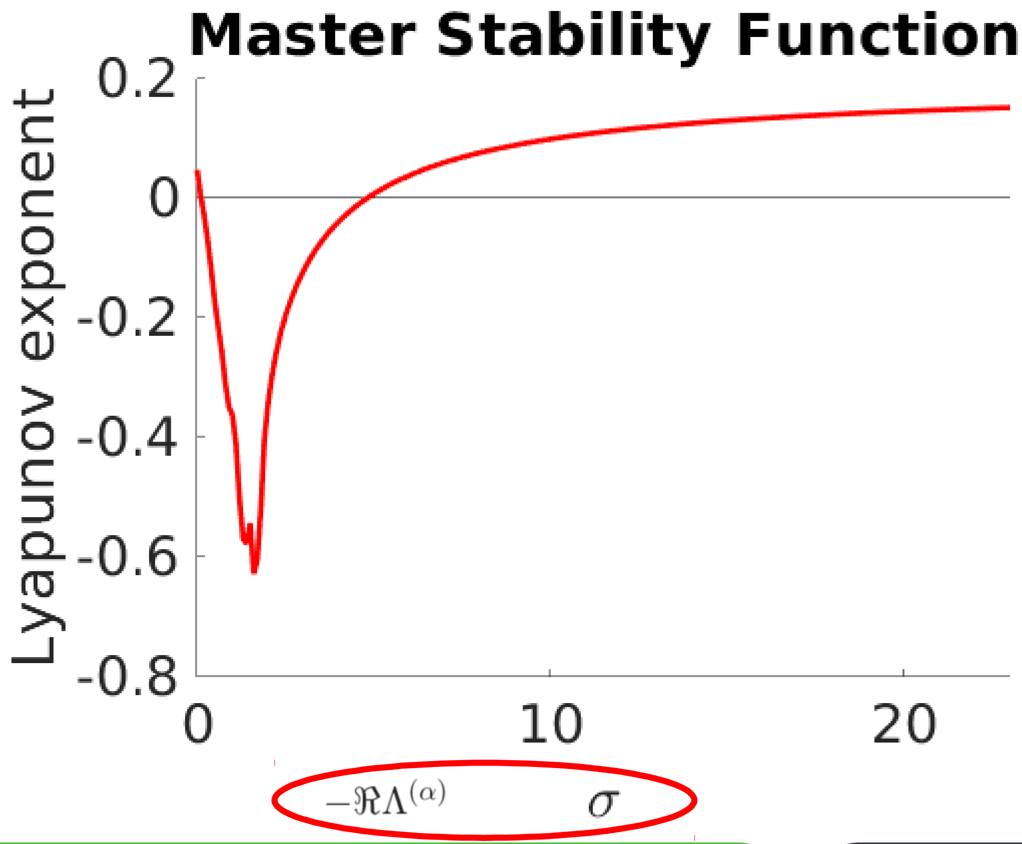

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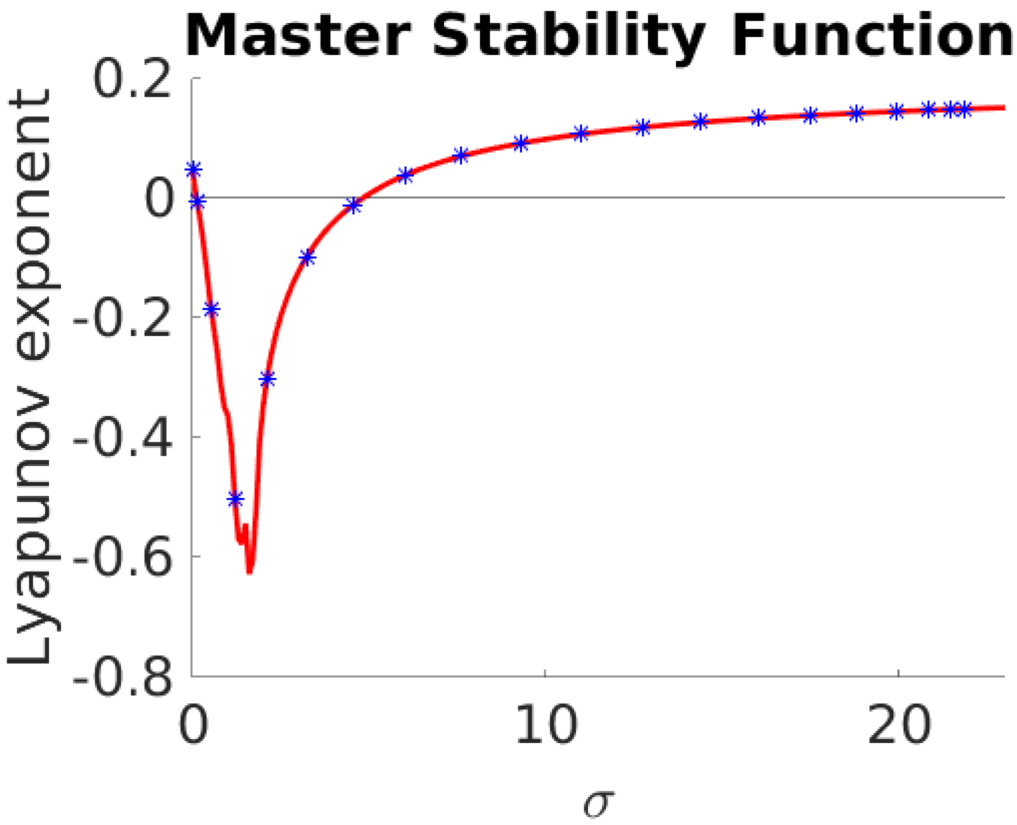

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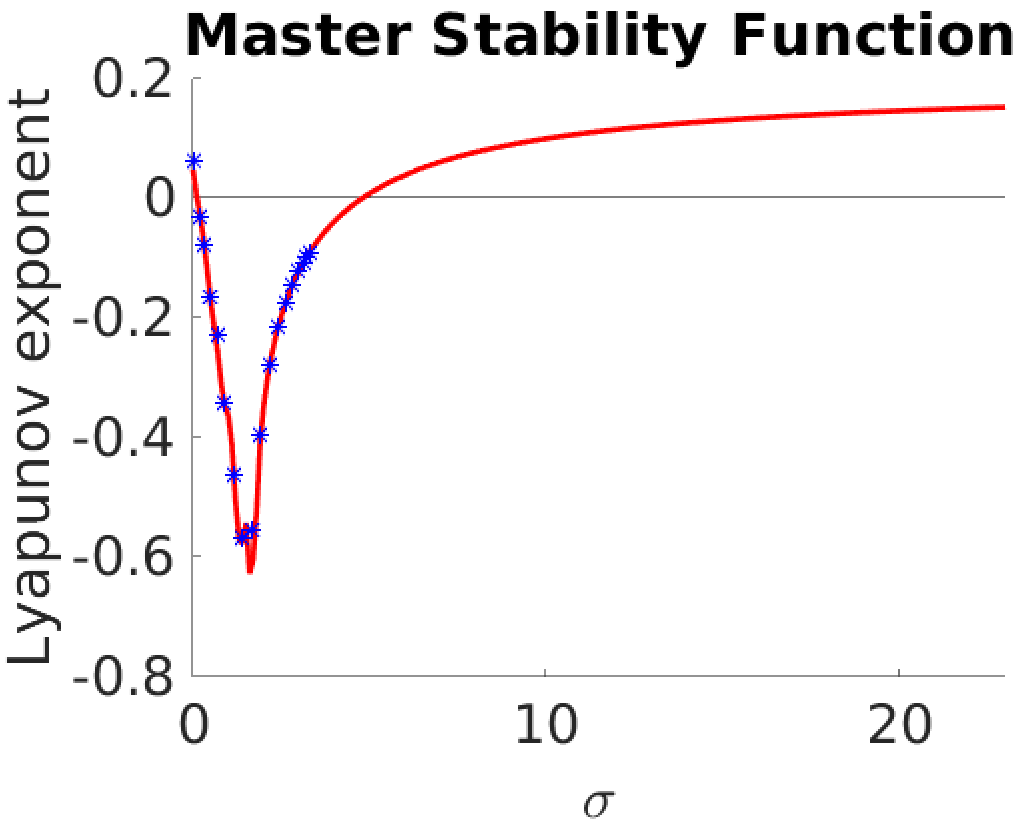
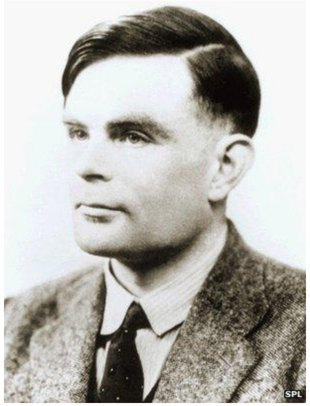


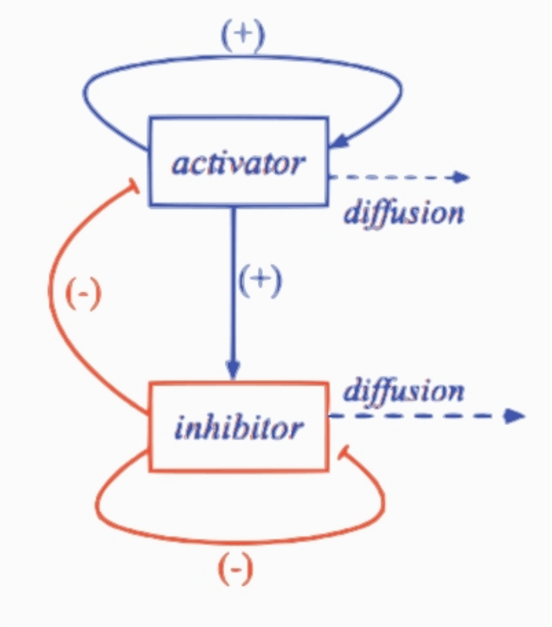
Intermezzo: Turing theory



The Chemical Basis of Morphogenesis

A. M. Turing

Philosophical Transactions of the Royal Society of London. Series B, Biological Sciences, Vol. 237, No. 641. (Aug. 14, 1952), pp. 37-72.



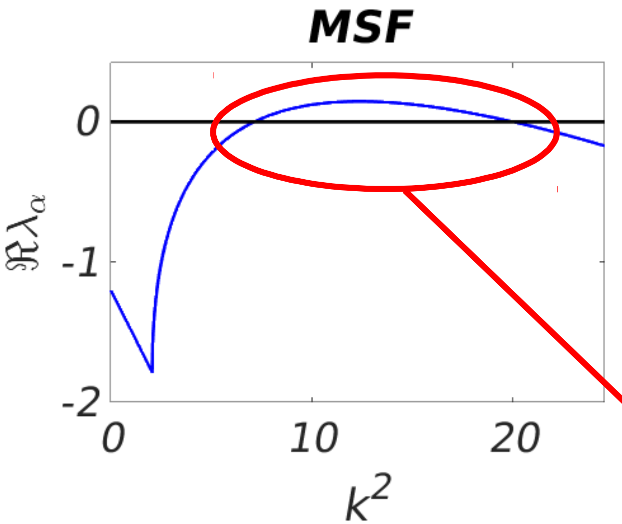
$$\begin{cases} \dfrac{\partial u}{\partial t}(x,t) = f(u,v) + D_u \nabla^2 u(x,t) \\ \\ \dfrac{\partial v}{\partial t}(x,t) = g(u,v) + D_v \nabla^2 v(x,t) \end{cases}$$

+ boundary conditions

+ domain of the Laplacian

+ ….

Turing theory in a nutshell



unstable modes

reaction-diffusion system of two species
(activator *u* and inhibitor *v*)

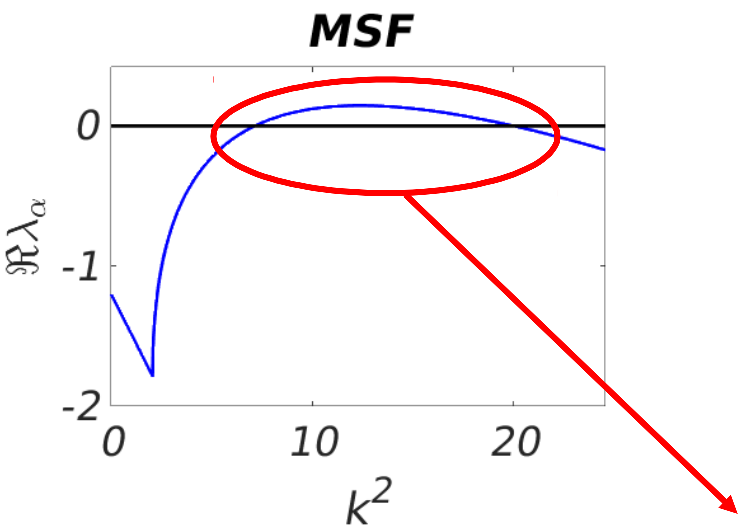
homogeneous stable state (fixed point)

inhomogeneous perturbations

exponential instability (diffusion-driven)

patterns $D_v > D_u$

Turing theory in a nutshell



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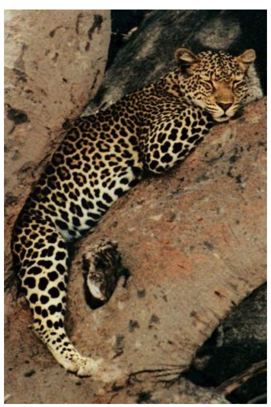

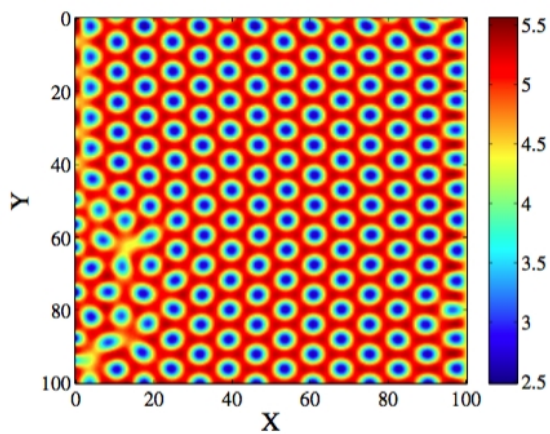

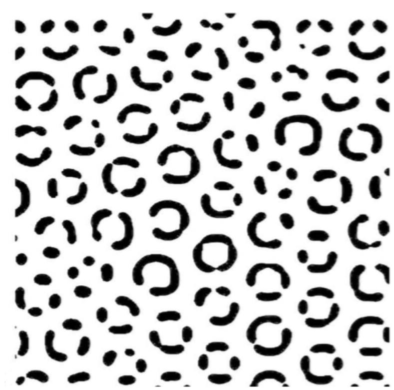

Many applications

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RESEARCH ARTICLE

A dot-stripe Turing model of joint patterning in the tetrapod limb

Jake Cornwall Scoones[1] and Tom W. Hiscock[2,3,*]

Article | Published: 08 July 2021

Nanoscale Turing patterns in a bismuth monolayer

Yuki Fuseya, Hiroyasu Katsuno, Kamran Behnia & Aharon Kapitulnik

Nature Physics **17**, 1031–1036 (2021) | Cite this article

PHYSICAL REVIEW RESEARCH **3**, 023241 (2021)

Linguistic evolution driven by network heterogeneity and the Turing mechanism

Sayat Mimar,[1] Mariamo Mussa Juane,[2] Jorge Mira,[3] Juyong Park,[4] Alberto P. Muñuzuri,[2] and Gourab Ghoshal[1,*]
[1]*Department of Physics & Astronomy, University of Rochester, Rochester, New York 14607, USA*
[2]*Group of Nonlinear Physics. Universidade de Santiago de Compostela, 15782 Santiago de Compostela, Spain*
[3]*Departamento de Física Aplicada. Universidade de Santiago de Compostela, 15782 Santiago de Compostela, Spain*
[4]*Graduate School of Culture Technology, Korea Advanced Institute of Science and Technology, Daejon, 305-701, Korea*

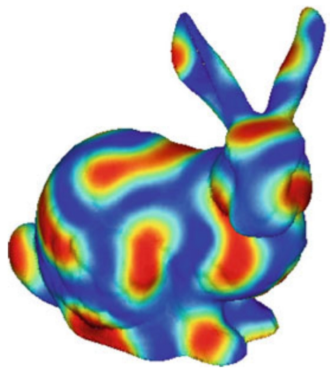
scientific reports

OPEN

Turing instability in quantum activator–inhibitor systems

Yuzuru Kato[1] & Hiroya Nakao[2]

Many applications



 | Development (2020) 147, dev183699. doi:10.1242/dev.183699

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[4]*Graduate School of Culture Technology, Korea Advanced Institute of Science and Technology, Daejon, 305-701, Korea*

scientific reports

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Turing instability in quantum activator–inhibitor systems

Yuzuru Kato[1] & Hiroya Nakao[2]

Extension on networks

ARTICLES
PUBLISHED ONLINE: 25 APRIL 2010 | DOI: 10.1038/NPHYS1651
nature physics

Turing patterns in network-organized activator–inhibitor systems

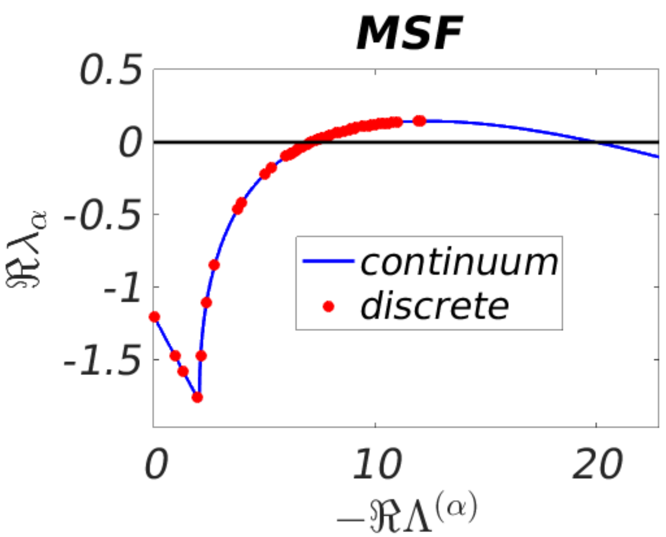
Hiroya Nakao[1,2]* and Alexander S. Mikhailov[3]*

J. theor. Biol. (1971) **32**, 507–537

Instability and Dynamic Pattern in Cellular Networks

H. G. Othmer† and L. E. Scriven‡

Department of Chemical Engineering and Materials Science Institute of Technology, University of Minnesota, Minneapolis, Minnesota 55455, *U.S.A.*



$$L_{ij} = A_{ij} - k_i \delta ij$$

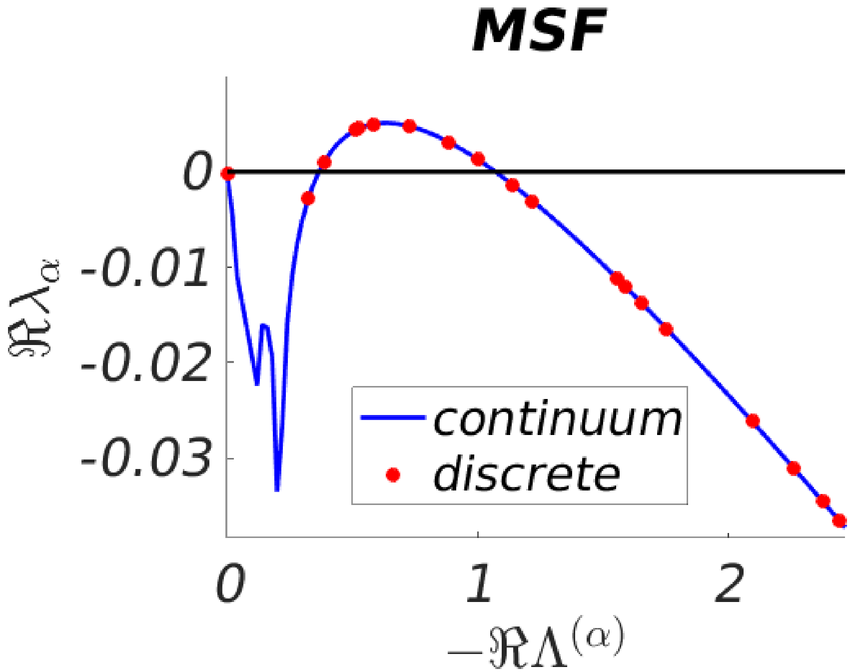
$$\begin{cases} \dfrac{du_i}{dt} = f(u_i, v_i) + D_u \displaystyle\sum_{j=1}^{nodes} L_{ij} u_j \\ \dfrac{dv_i}{dt} = g(u_i, v_i) + D_v \displaystyle\sum_{j=1}^{nodes} L_{ij} v_j \end{cases}$$

Turing-like instability

PHYSICAL REVIEW E **92**, 022818 (2015)

Turing-like instabilities from a limit cycle

Joseph D. Challenger,[1,2] Raffaella Burioni,[3] and Duccio Fanelli[2]



the homogeneous stable state is a **limit cycle**

Coupled oscillators on networks (!)

$$\dot{\vec{x}}_i = \vec{f}(\vec{x}_i) + \sigma_1 \sum_j A_{ij}^{(1)} \vec{g}^{(1)}(\vec{x}_i, \vec{x}_j)$$

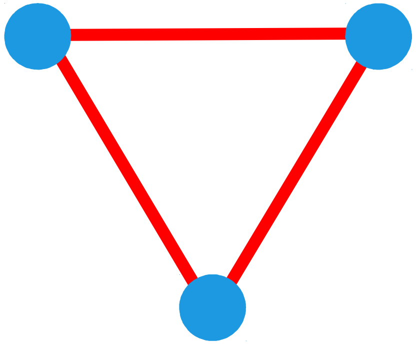
dynamics of x_i

chaotic, oscillating,

Fixed point, etc...

pairwise coupling

in general diffusive like



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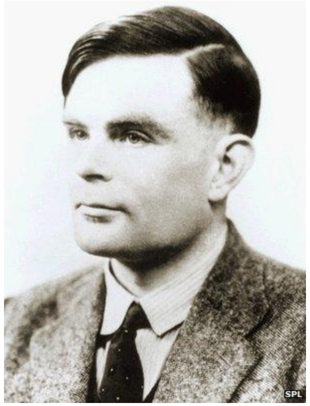

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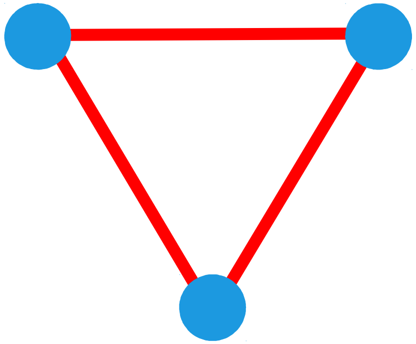

pairwise coupling

in general diffusive like

Extensions of Turing theory

PHYSICAL REVIEW E **92**, 022818 (2015)

Turing-like instabilities from a limit cycle

Joseph D. Challenger,[1,2] Raffaella Burioni,[3] and Duccio Fanelli[2]

Journal of Theoretical Biology 480 (2019) 81–91

Contents lists available at ScienceDirect

Journal of Theoretical Biology

journal homepage: www.elsevier.com/locate/jtb

Patterns of non-normality in networked systems

Riccardo Muolo[a], Malbor Asllani[b,c,*], Duccio Fanelli[d], Philip K. Maini[b], Timoteo Carletti[e]

nature COMMUNICATIONS

ARTICLE

Received 5 Feb 2014 | Accepted 26 Jun 2014 | Published 31 Jul 2014 DOI: 10.1038/ncomms5517

The theory of pattern formation on directed networks

Malbor Asllani[1,2], Joseph D. Challenger[2], Francesco Saverio Pavone[2,3,4], Leonardo Sacconi[3,4] & Duccio Fanelli[2]

Journal of Physics: Complexity

OPEN ACCESS

PAPER

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Finite propagation enhances Turing patterns in reaction–diffusion networked systems

Timoteo Carletti* and Riccardo Muolo

Department of Mathematics & naXys, Namur Institute for Complex Systems, University of Namur, rue Grafé 2, 5000 Namur, Belgium

* Author to whom any correspondence should be addressed.

E-mail: timoteo.carletti@unamur.be

Chaos, Solitons and Fractals 134 (2020) 109707

Generalized patterns from local and non local reactions

Giulia Cencetti[a], Federico Battiston[b], Timoteo Carletti[c], Duccio Fanelli[d,*]

Directed Networks

nature COMMUNICATIONS

ARTICLE

Received 5 Feb 2014 | Accepted 26 Jun 2014 | Published 31 Jul 2014 DOI: 10.1038/ncomms5517

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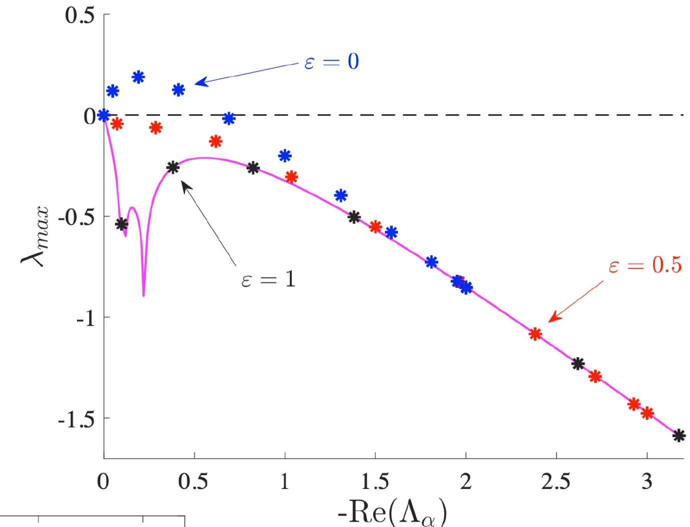
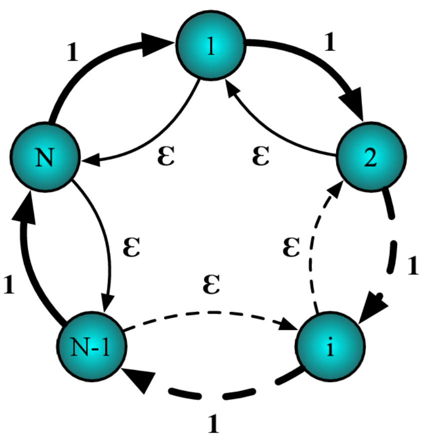
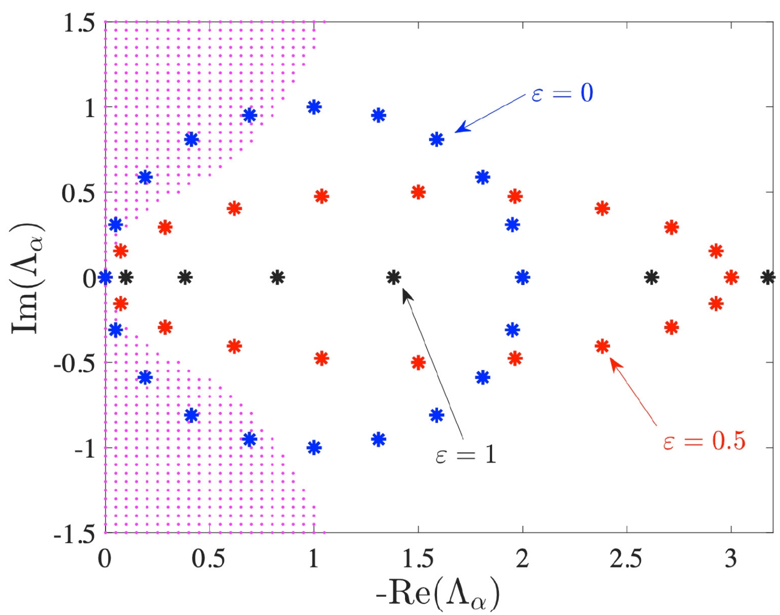






Non-normal Networks

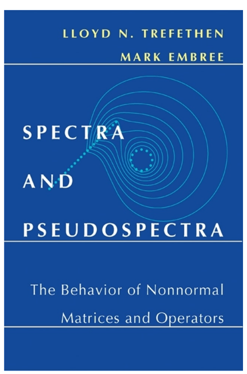
A network is non-normal
when $A^*A \neq AA^*$

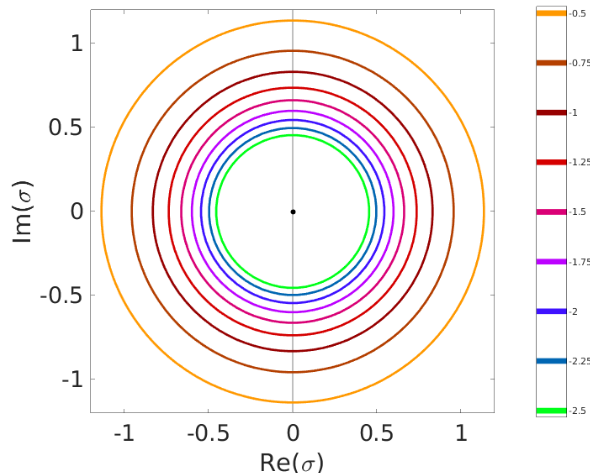
RESEARCH ARTICLE | NETWORK SCIENCE

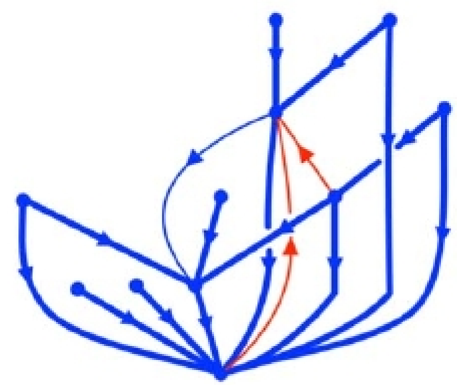
Structure and dynamical behavior of non-normal networks

Malbor Asllani[1,2], Renaud Lambiotte[1] and Timoteo Carletti[2,*]
+ See all authors and affiliations

Science Advances 12 Dec 2018:
Vol. 4, no. 12, eaau9403
DOI: 10.1126/sciadv.aau9403





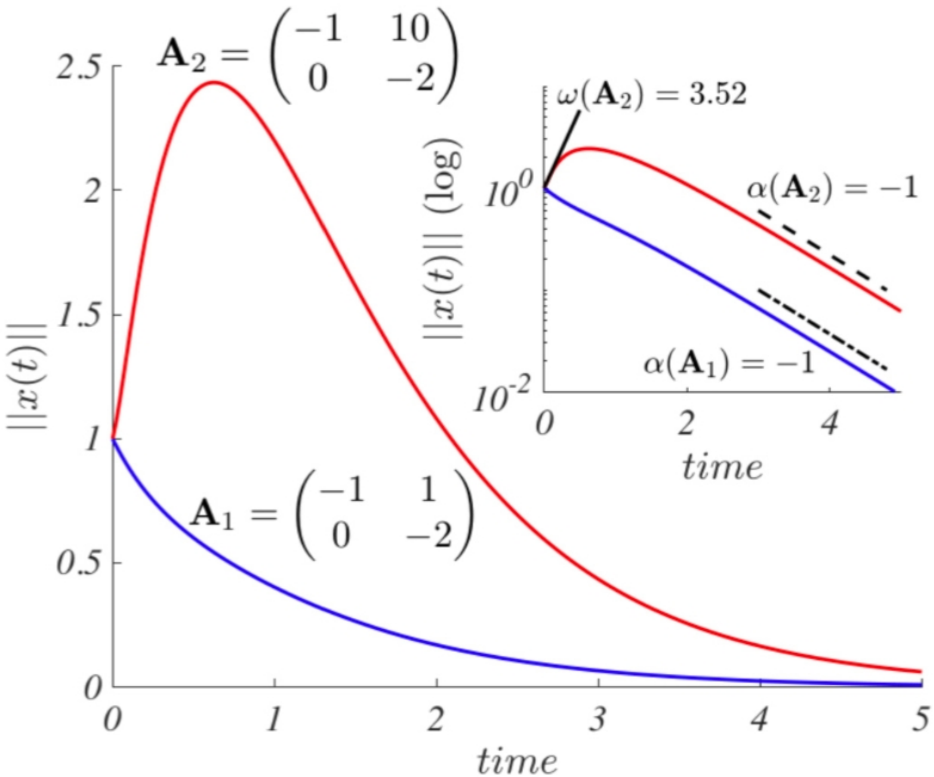


$$\beta\text{-pseudospectrum } \lambda_\beta(A) \rightarrow \lambda(A+B) \; ||B|| < \beta$$

Effects on the dynamics

ω largest eigenvalue of the hermitian (simmetric) part

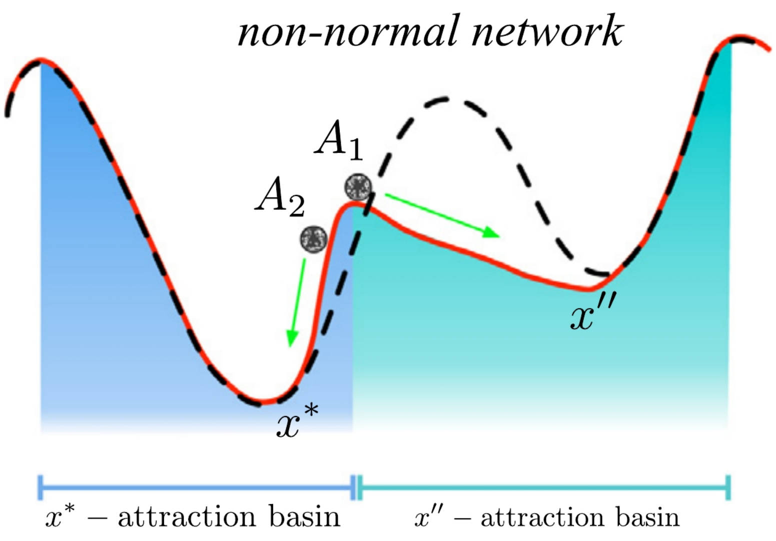
Linear dynamics

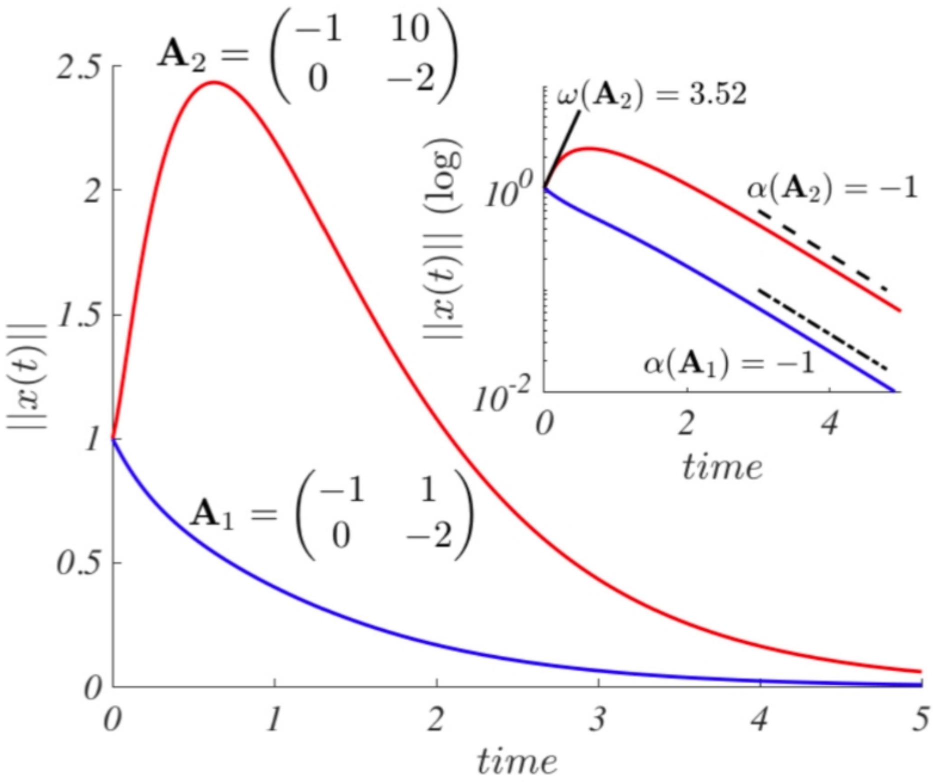


Effects on the dynamics

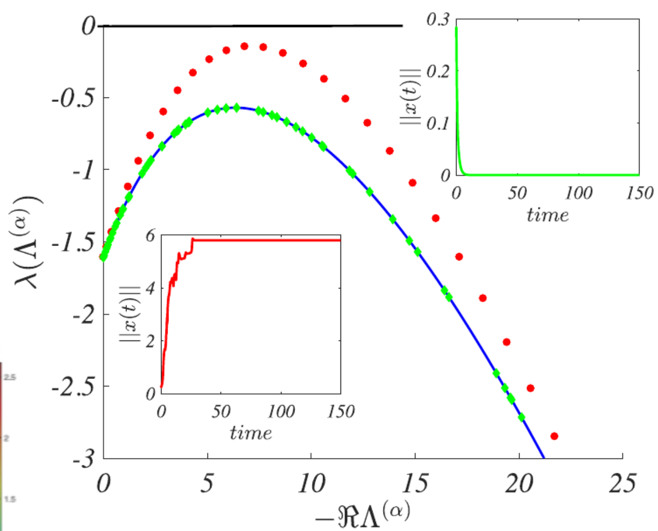
Linear dynamics

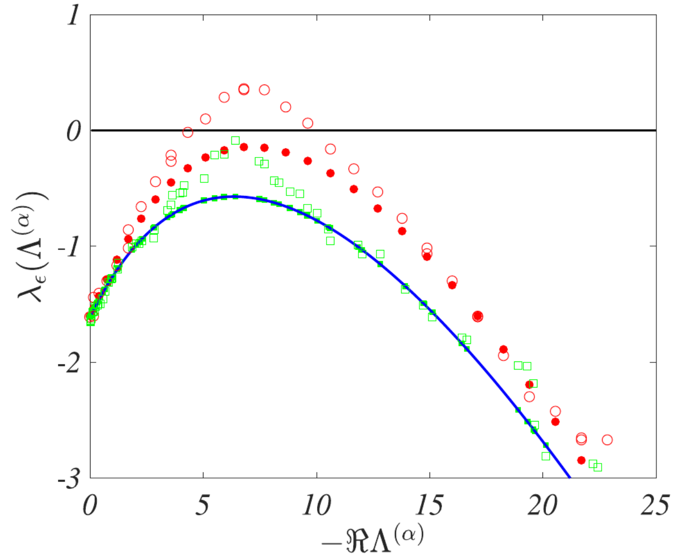
Non-linear dynamics

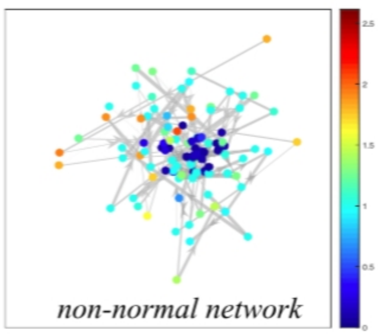


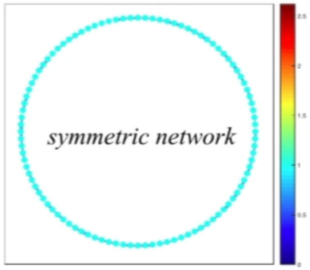


Patterns of Non-normality









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Patterns of non-normality in networked systems

Riccardo Muolo[a], Malbor Asllani[b,c,*], Duccio Fanelli[d], Philip K. Maini[b], Timoteo Carletti[e]

Synchronization and non-normality

entropy

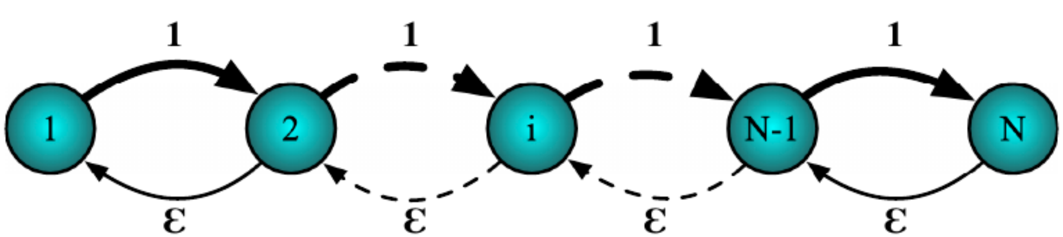
Entropy **2021**, *23*, 36.

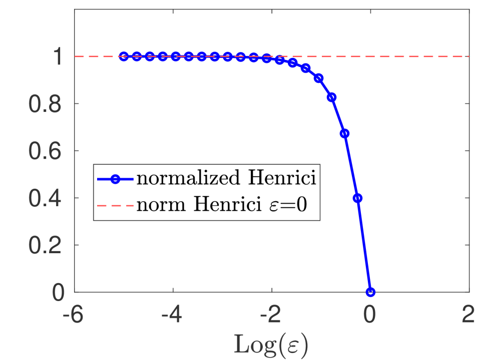
MDPI

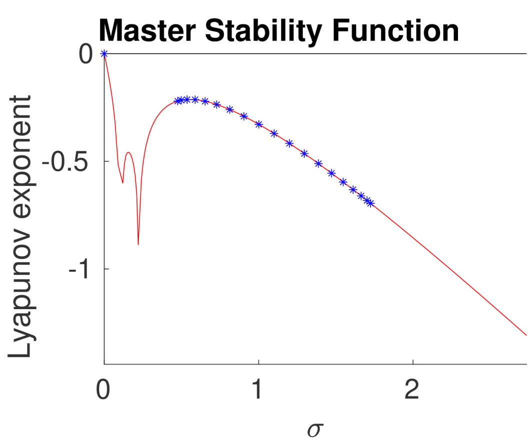
Article

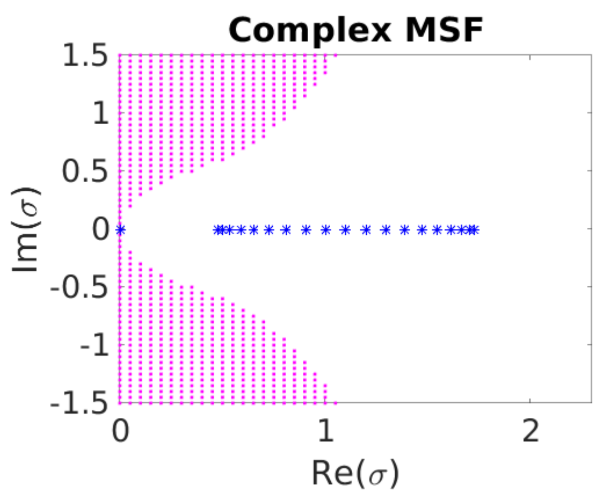
Synchronization Dynamics in Non-Normal Networks: The Trade-Off for Optimality

Riccardo Muolo [1,*], Timoteo Carletti [1], James P. Gleeson [2] and Malbor Asllani [1,2]

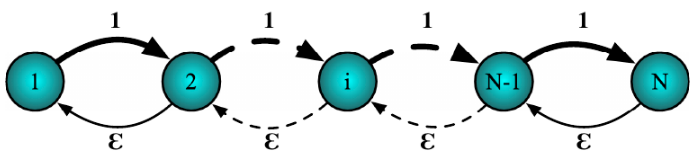


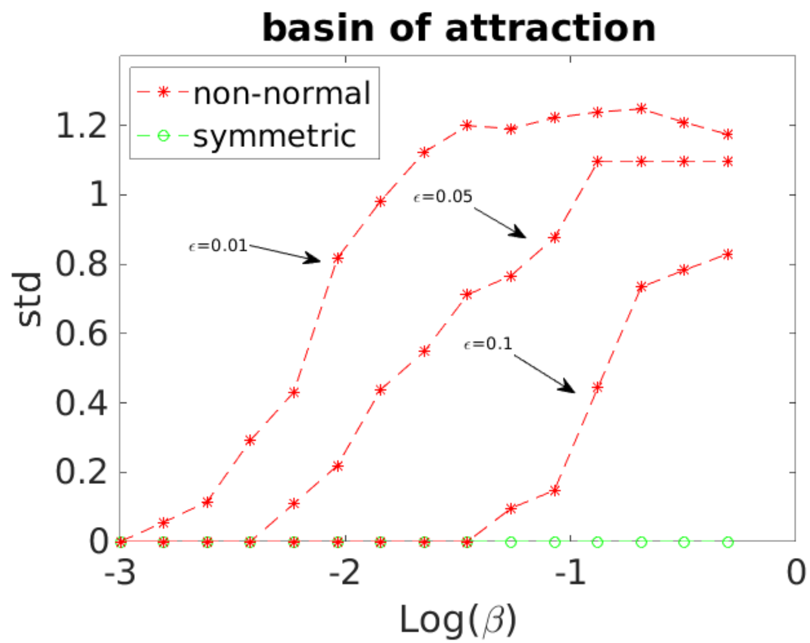


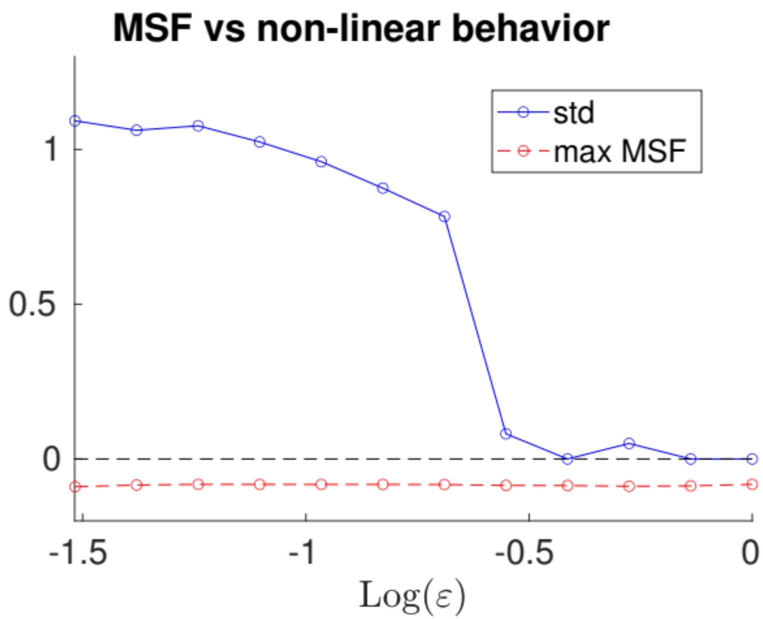


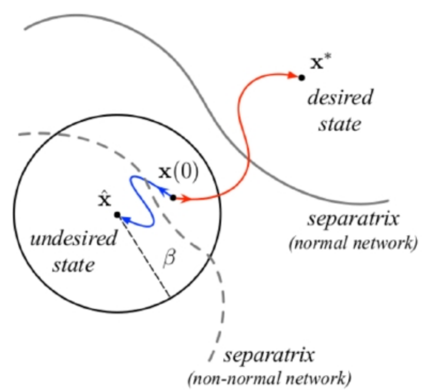


De-synchronization









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Debate on optimal networks

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Article

Synchronization Dynamics in Non-Normal Networks: The Trade-Off for Optimality

Riccardo Muolo [1,*], Timoteo Carletti [1], James P. Gleeson [2] and Malbor Asllani [1,2]

Comment on "Synchronization dynamics in non-normal networks: the trade-off for optimality"

Takashi Nishikawa,[1,2,*] Adilson E. Motter,[1,2] and Louis M. Pecora[3]

Reply to Comment on "Synchronization dynamics in non-normal networks: the trade-off for optimality"

Riccardo Muolo[1], Timoteo Carletti[1], James P. Gleeson[2], Malbor Asllani[3]

Comment on "Synchronization dynamics in non-normal networks: the trade-off for optimality"

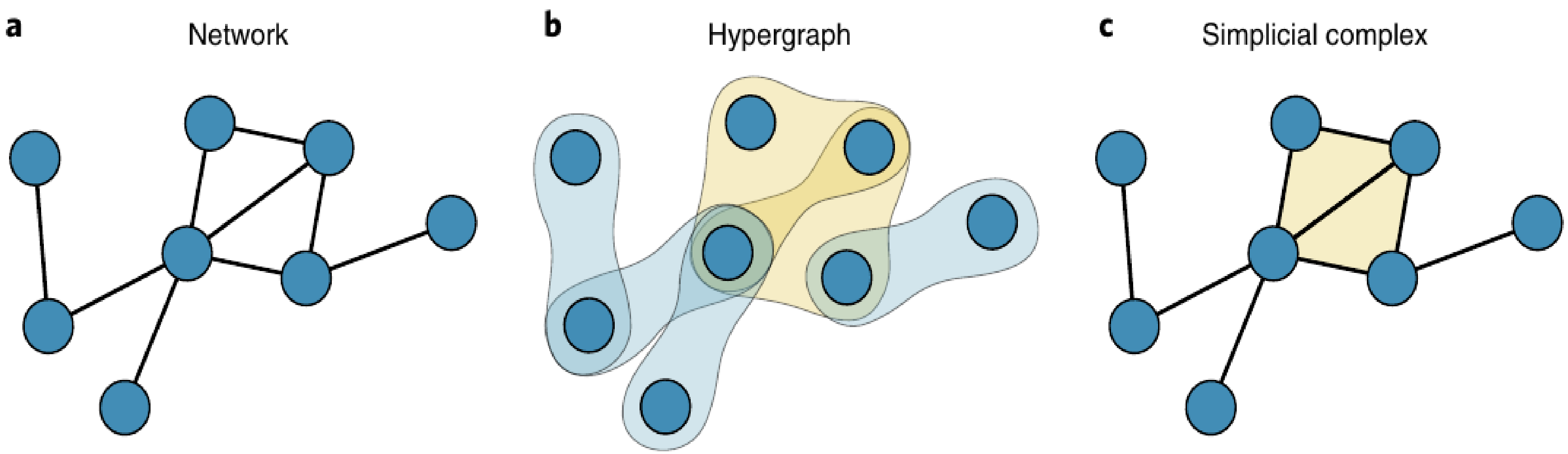
Francesco Sorrentino and Chad Nathe

Non-normality, optimality and synchronization

Jeremie Fish*

Erik M. Bollt

High-order Structures

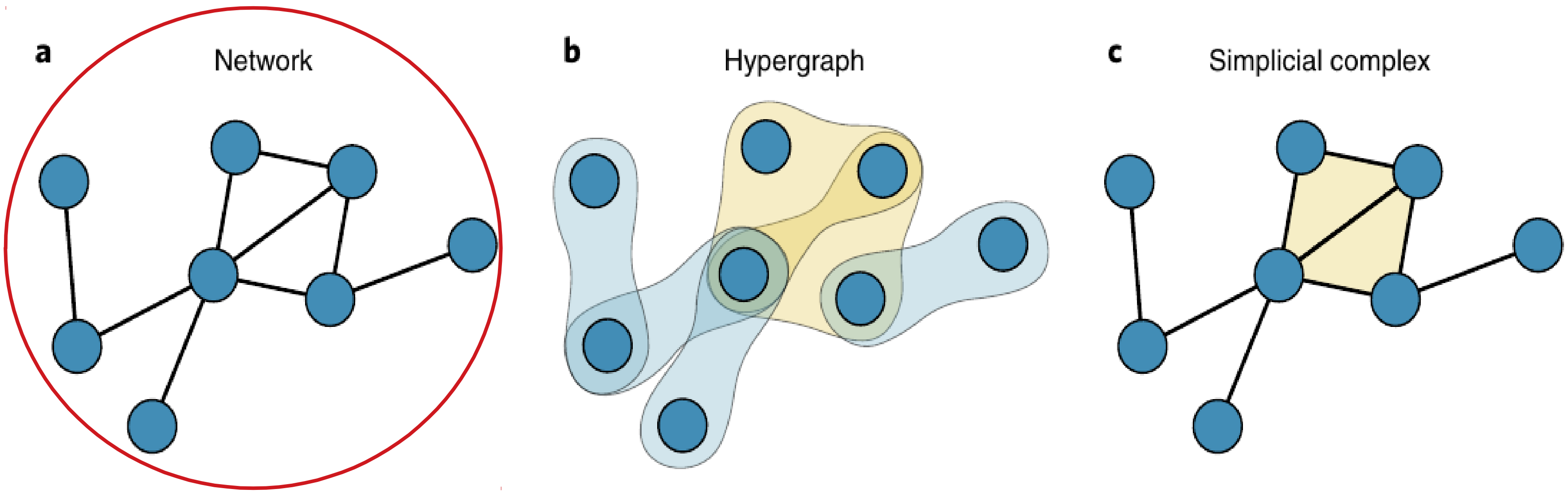


Battiston et al., Nat. Phys., 2021

High-order Structures

adjacency matrix

adjacency tensors



Battiston et al., Nat. Phys., 2021

High-order Structures

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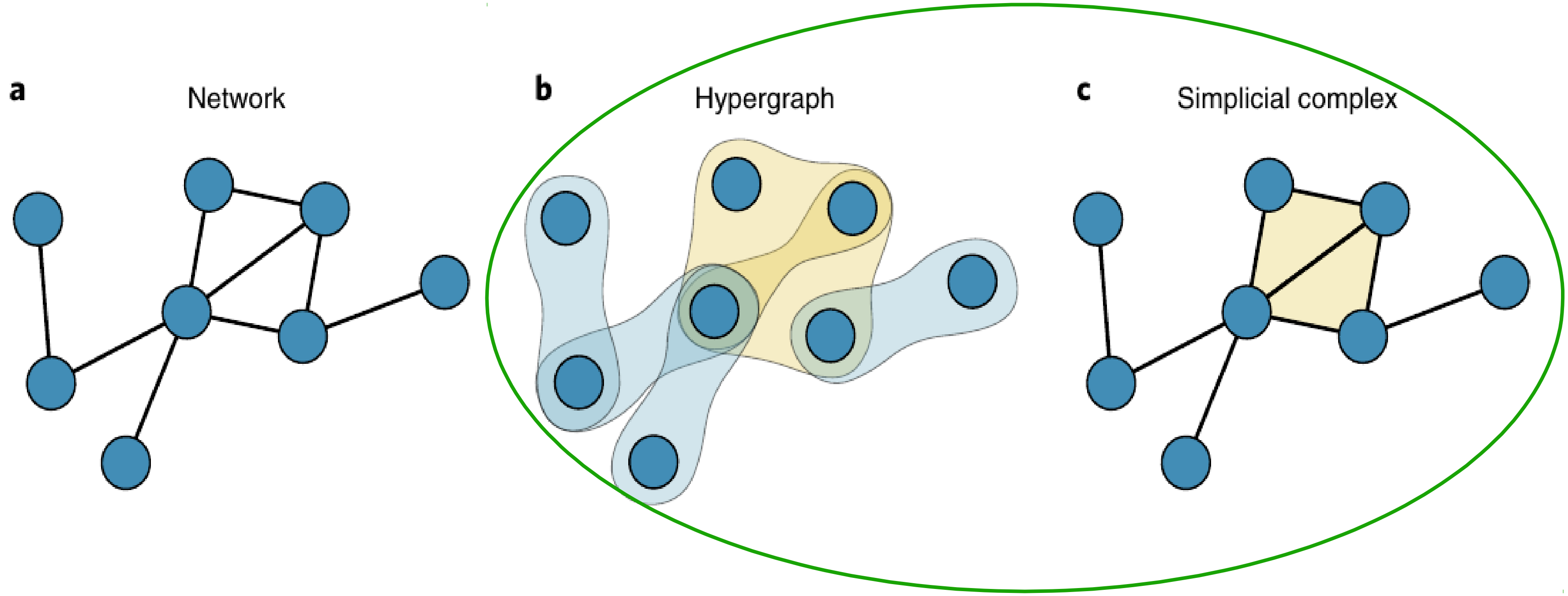


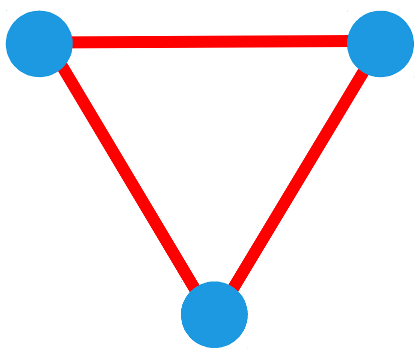
Battiston et al., Nat. Phys., 2021

High-order coupled oscillators

$$\dot{\vec{x}}_i = \vec{f}(\vec{x}_i) + \sigma_1 \sum_j A^{(1)}_{ij} \vec{g}^{(1)}(\vec{x}_i, \vec{x}_j)$$

dynamics of x_i

pairwise coupling

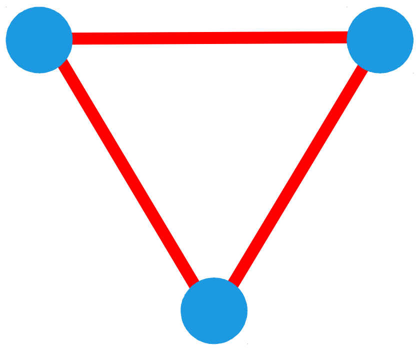


High-order coupled oscillators

$$\dot{\vec{x}}_i = \vec{f}(\vec{x}_i) + \sigma_1 \sum_j A^{(1)}_{ij} \vec{g}^{(1)}(\vec{x}_i, \vec{x}_j) + \sigma_2 \sum_{j,k} A^{(2)}_{ijk} \vec{g}^{(2)}(\vec{x}_i, \vec{x}_j, \vec{x}_k)$$

dynamics of x_i

pairwise coupling

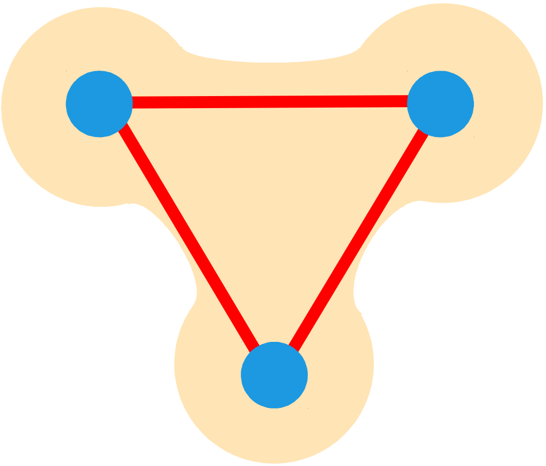


High-order coupled oscillators

$$\dot{\vec{x}}_i = \vec{f}(\vec{x}_i) + \sigma_1 \sum_j A_{ij}^{(1)} \vec{g}^{(1)}(\vec{x}_i, \vec{x}_j) + \sigma_2 \sum_{j,k} A_{ijk}^{(2)} \vec{g}^{(2)}(\vec{x}_i, \vec{x}_j, \vec{x}_k)$$

dynamics of x_i

high-order
(3-body) coupling



pairwise coupling

Dynamics on high-order structures

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https://doi.org/10.1038/s41467-021-21486-9 OPEN

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Stability of synchronization in simplicial complexes

L. V. Gambuzza[1,12], F. Di Patti[2,12], L. Gallo[3,4,12], S. Lepri[2], M. Romance[5], R. Criado[5], M. Frasca[1,6,13]✉, V. Latora[3,4,7,8,13]✉ & S. Boccaletti[2,9,10,11,13]✉

J.Phys.Complex. 1 (2020) 035006 (16pp)

Journal of Physics: Complexity

PAPER

Dynamical systems on hypergraphs

Timoteo Carletti[1,4], Duccio Fanelli[2] and Sara Nicoletti[2,3]

Turing theory in high-order systems

arXiv > nlin > arXiv:2207.03985

Nonlinear Sciences > Pattern Formation and Solitons

[Submitted on 8 Jul 2022]

Turing patterns in systems with high-order interactions

Riccardo Muolo, Luca Gallo, Vito Latora, Mattia Frasca, Timoteo Carletti

$$\dot{u}_i = f_1(u_i, v_i) + \sigma_1 D_u^{(1)} \sum_{j_1=1}^{N} A_{ij_1}^{(1)}\left(h_1^{(1)}(u_{j_1}) - h_1^{(1)}(u_i)\right) + \sigma_2 D_u^{(2)} \sum_{j_1=1}^{N}\sum_{j_2=1}^{N} A_{ij_1j_2}^{(2)}\left(h_1^{(2)}(u_{j_1}, u_{j_2}) - h_1^{(2)}(u_i, u_i)\right)$$

$$\dot{v}_i = f_2(u_i, v_i) + \sigma_1 D_v^{(1)} \sum_{j_1=1}^{N} A_{ij_1}^{(1)}\left(h_2^{(1)}(v_{j_1}) - h_2^{(1)}(v_i)\right) + \sigma_2 D_v^{(2)} \sum_{j_1=1}^{N}\sum_{j_2=1}^{N} A_{ij_1j_2}^{(2)}\left(h_2^{(2)}(v_{j_1}, v_{j_2}) - h_2^{(2)}(v_i, v_i)\right)$$

Natural coupling

$$\frac{d}{dt}\vec{\xi} = \left(\mathbb{I}_N \otimes \mathbf{J}_0 + \sigma_1 \mathbf{L}^{(1)} \otimes \mathbf{J}_{H^{(1)}} + \sigma_2 \mathbf{L}^{(2)} \otimes \mathbf{J}_{H^{(2)}}\right)\vec{\xi}$$

Natural coupling

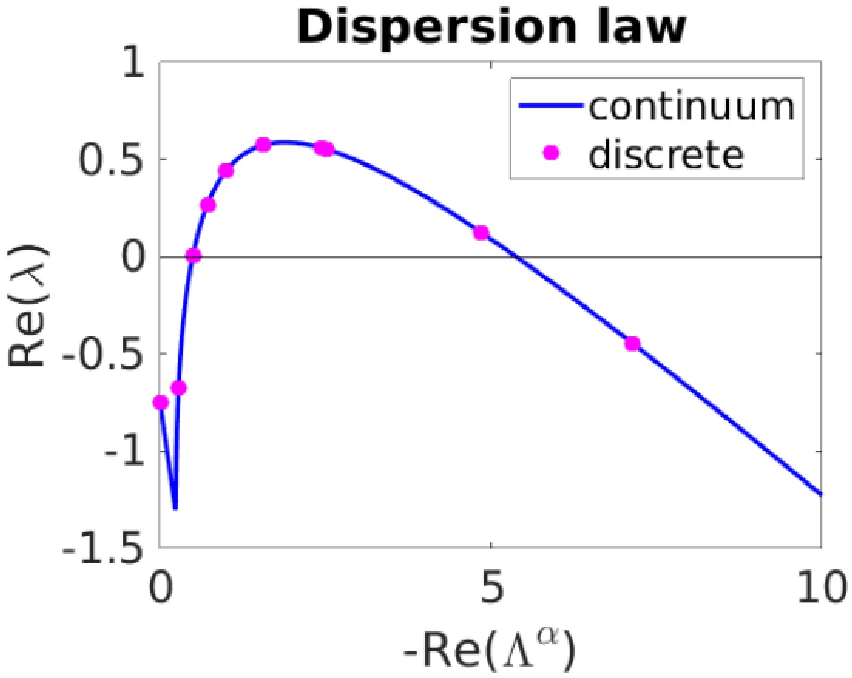
$$\frac{d}{dt}\vec{\xi} = \left(\mathbb{I}_N \otimes \mathbf{J}_0 + \sigma_1 \mathbf{L}^{(1)} \otimes \mathbf{J}_{H^{(1)}} + \sigma_2 \mathbf{L}^{(2)} \otimes \mathbf{J}_{H^{(2)}}\right)\vec{\xi}$$

$$\vec{h}^{(d)}(\vec{x}, \ldots, \vec{x}) = \ldots = \vec{h}^{(2)}(\vec{x}, \vec{x}) = \vec{h}^{(1)}(\vec{x})$$

same diffusion coefficients
for every order

$$\frac{d}{dt}\vec{\xi} = \left[\mathbb{I}_N \otimes \mathbf{J}_0 + \left(\sigma_1 \mathbf{L}^{(1)} + \sigma_2 \mathbf{L}^{(2)}\right) \otimes \mathbf{J}_{H^{(1)}}\right]\vec{\xi}$$

Natural coupling



same diffusion coefficients
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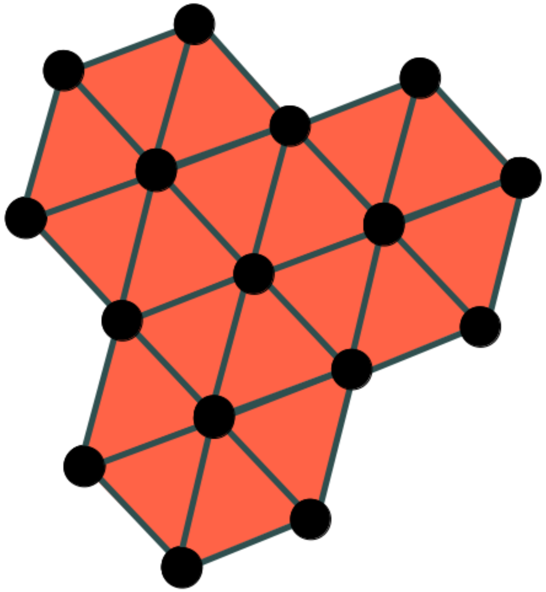
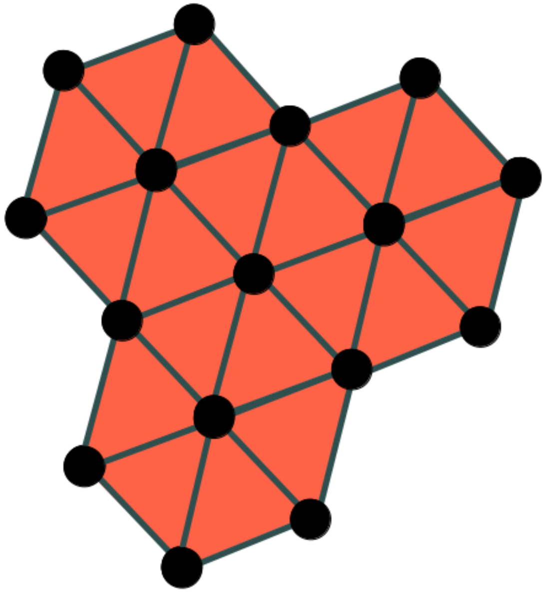
$$\frac{d}{dt}\vec{\xi} = \left[\mathbb{I}_N \otimes \mathbf{J}_0 + \left(\sigma_1 \mathbf{L}^{(1)} + \sigma_2 \mathbf{L}^{(2)}\right) \otimes \mathbf{J}_{H^{(1)}}\right]\vec{\xi}$$

Regular topologies

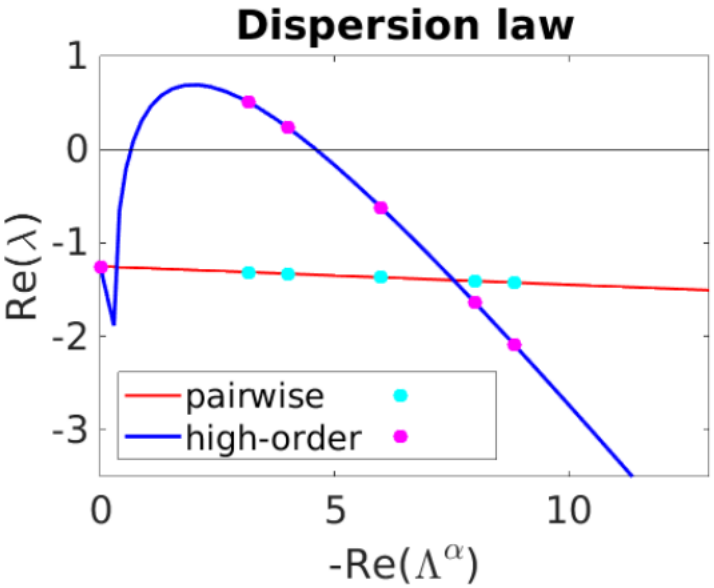


$$\mathbf{L}^{(2)} = 2\mathbf{L}^{(1)}$$

Regular topologies



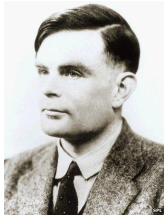
$$\mathbf{L}^{(2)} = 2\mathbf{L}^{(1)}$$



$$\frac{d}{dt}\vec{\xi} = \left(\mathbb{I}_N \otimes \mathbf{J}_0 + \mathbf{L}^{(1)} \otimes (\sigma_1 \mathbf{J}_{H^{(1)}} + 2\sigma_2 \mathbf{J}_{H^{(2)}})\right)\vec{\xi}$$

Thank you for your attention

Questions?



Take Home Messages

synchronization of coupled oscillators can be studied in the framework of Turing pattern formation (diffusive coupling)

the topology greatly affects the stability of the synchronized state and a linear stability analysis may fail in its predictions

the theory can be extended in the framework of high-order interactions

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