A lower bound for set colouring Ramsey numbers

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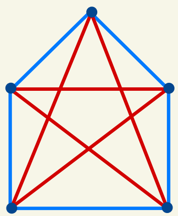
Probability Seminar - IM UFRJ

Ramsey number

$$R_r(k) = \min\{n \mid \forall\ c : E(K_n) \to [r] \text{ has a mono}^{x}\ K_k\}$$

ex.: $R_2(3)$



$R_2(3) > 5$

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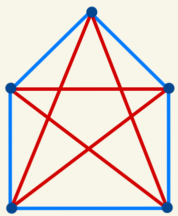

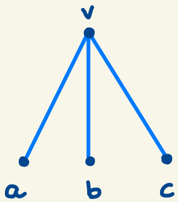

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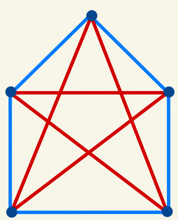

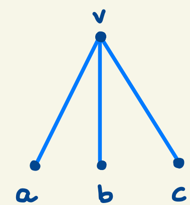

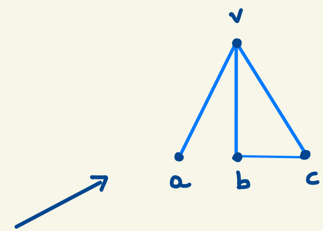

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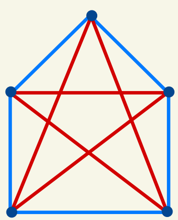

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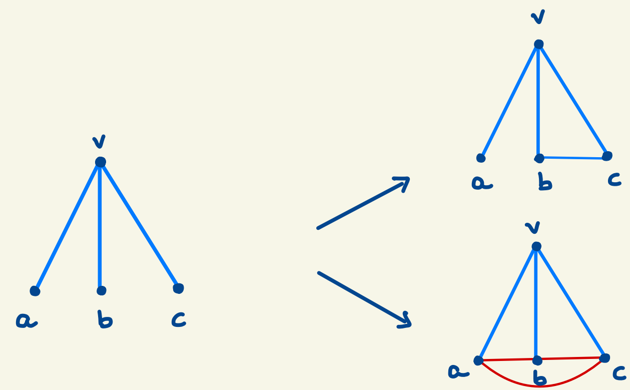

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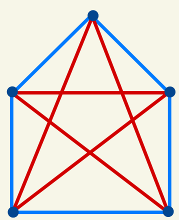
ex.: $R_2(3) = 6$



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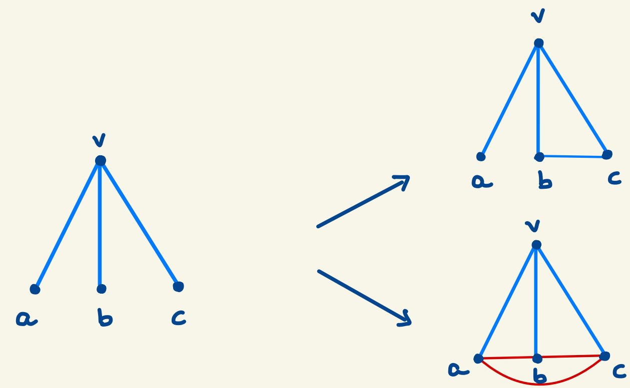

$R_2(3) \leq 6$

$\square$

Set colouring Ramsey

Set colouring Ramsey number

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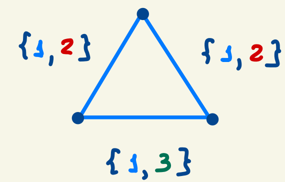
$\exists$ a common colour to all edges:



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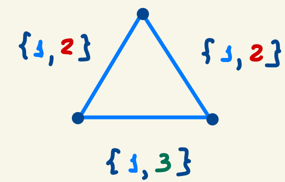

$s = 1$:

$$R_{r,1}(k) = R_r(k)$$

best known bounds

$$2^{\Omega(kr)} \leq R_r(k) \leq r^{O(kr)}$$

Ramsey bound

upper bound (Erdős - Szekeres) $R_{2,2}(k) \leq 4^k$

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↳ WTS: Any 2- colouring of $E(K_n)$ where $n = 4^k$ has a mono X clique.

proof:

Let c be a 2-colouring with no K_k mono X

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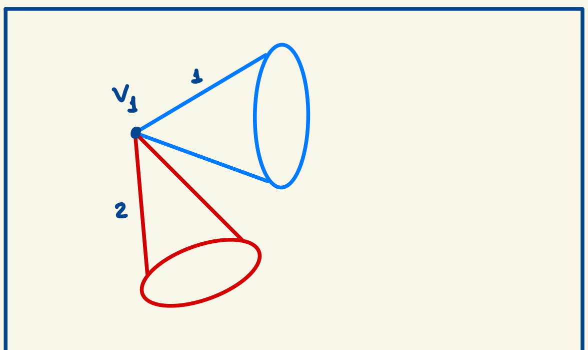

$X_0 = V$

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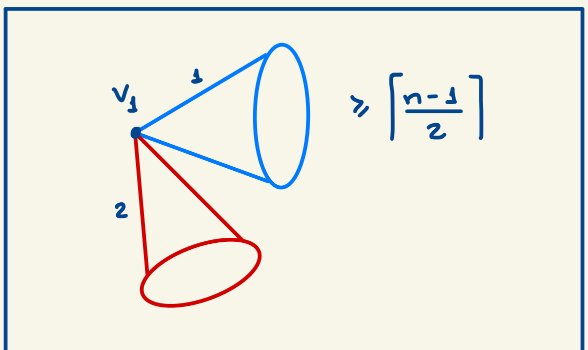

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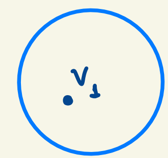

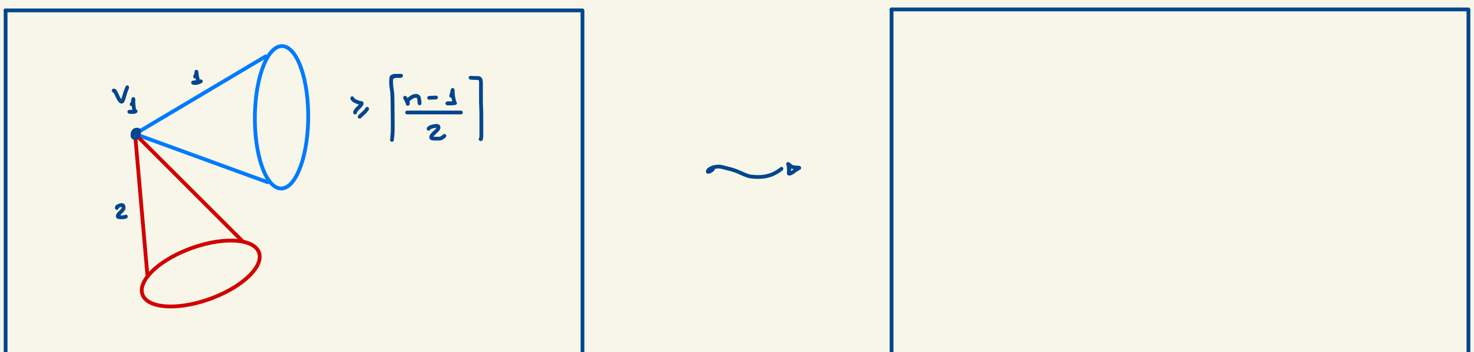

$X_0 = V$

$X_1 = X_0 \cap N_B(v_1)$

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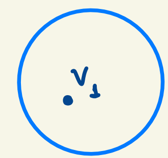

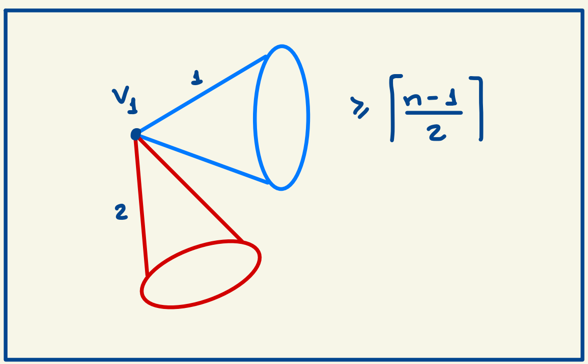

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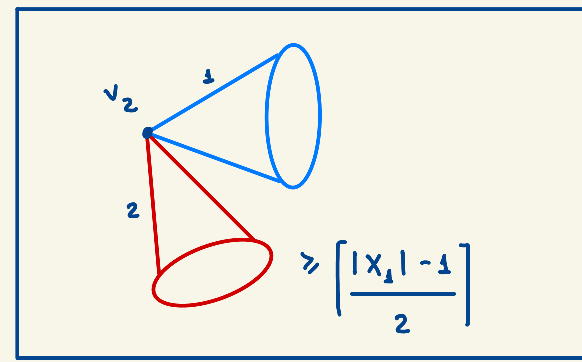
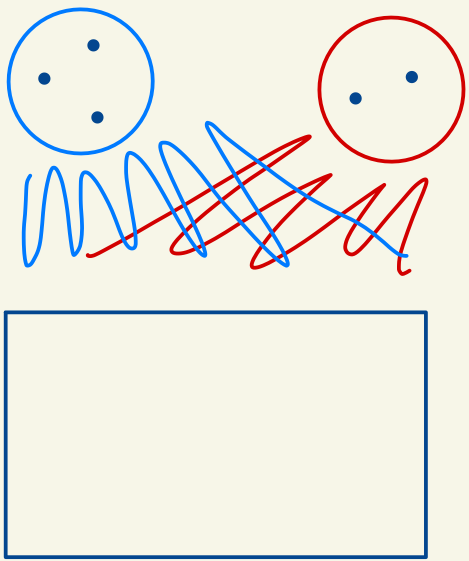


$X_1 = X_0 \cap N_B(v_1)$

Ramsey bound

upper bound (Erdős - Szekeres) $R_{2,1}(k) \le 4^k = 2^{2k}$



$$X_i = X_{i-1} \cap \bigcap_{v \in B} N_B(v) \cap \bigcap_{v \in R} N_R(v)$$

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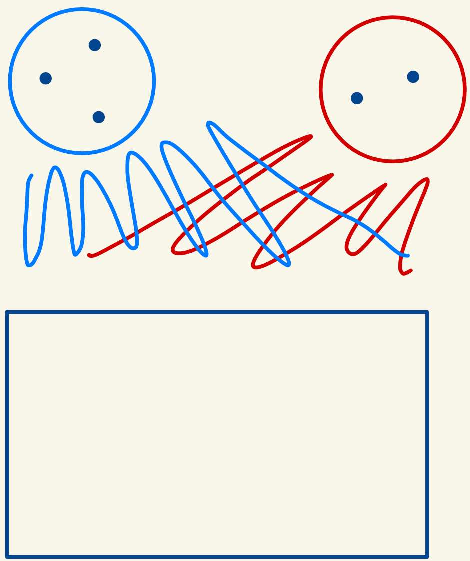

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$$|X_1| \geqslant (n-1)/2 = \frac{n}{2} - 1/2$$

$$|X_2| \geqslant (|X_2| - 1)/2 \geqslant \frac{n}{4} - 1/2 - 1/4$$

$$\vdots$$

$$|X_i| \geqslant \frac{n}{2^i} - 1$$

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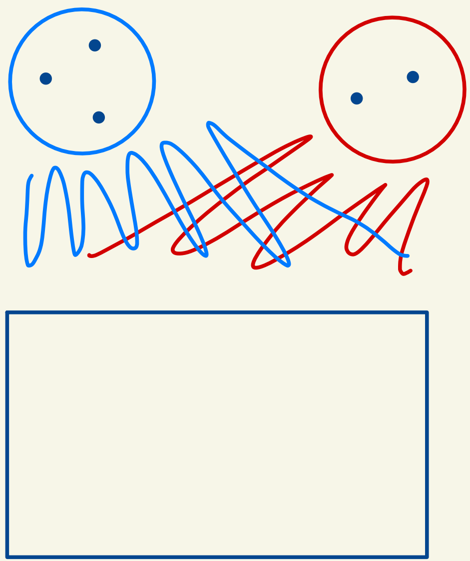

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when $i = 2(k-1)$,

$$|X_i| \geq \frac{4^k}{2^{2(k-1)}} - 1$$

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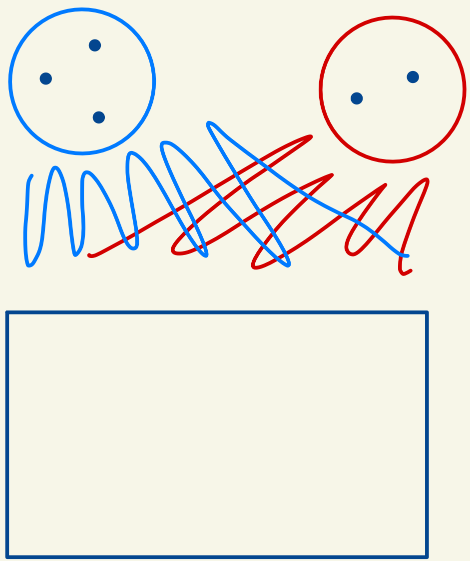

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we have a mono χ K_k!

$\square$

Ramsey bound

$R_2(k) \le (4-\varepsilon)^k$

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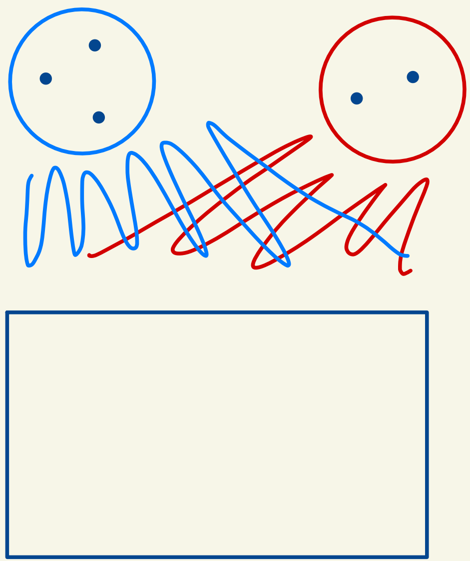

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upper bound (Erdős - Szekeres)

$$\boxed{R_{r,1}(k) \le r^{rk}}$$

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Set colouring Ramsey bounds

upper bound (Erdős - Szekeres) $R_{r,s}(k) = (r/s)^{rk}$

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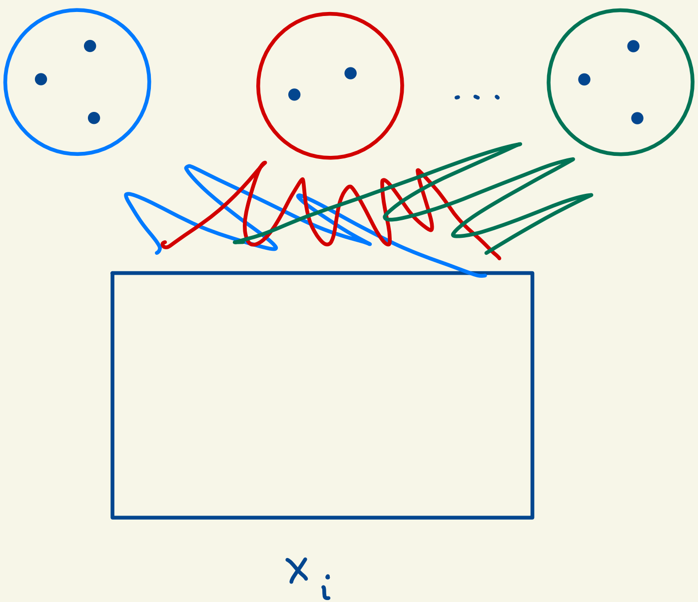

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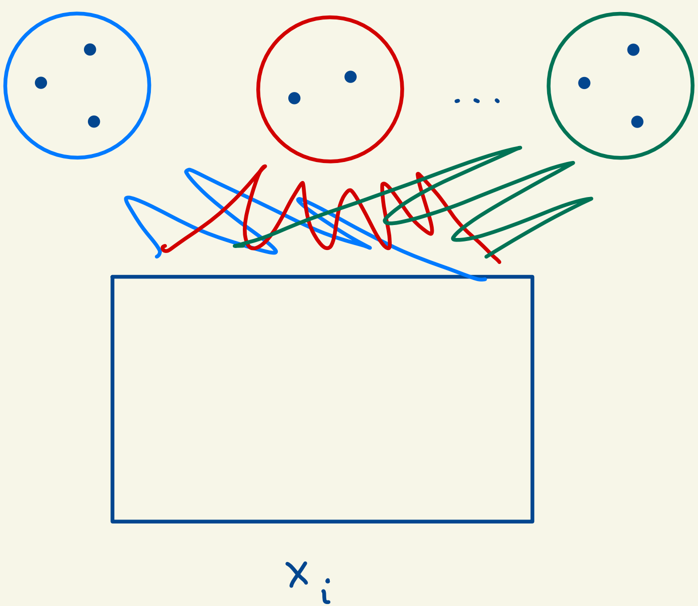

Given a vertex v in X_{i-1} :

majority color present in at least

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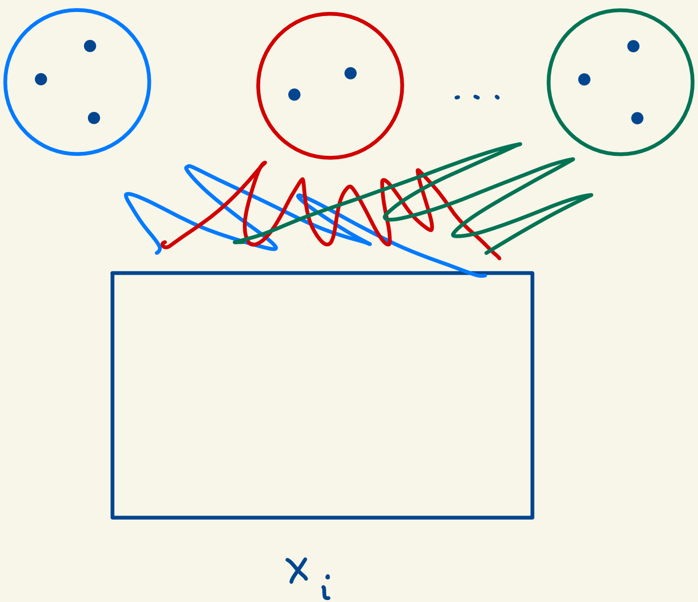

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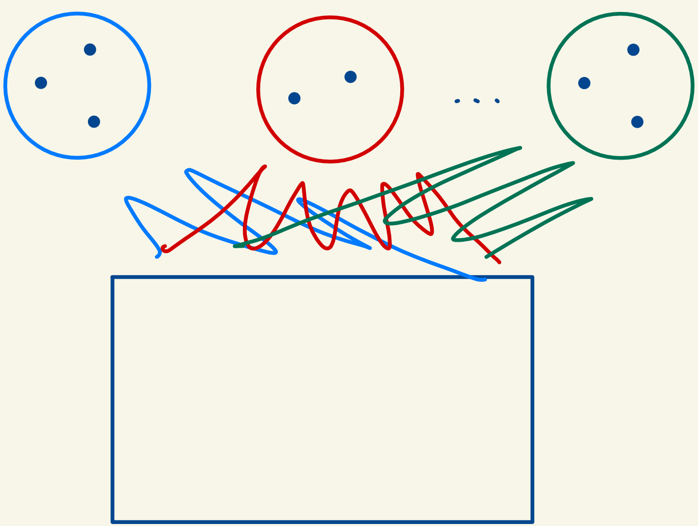

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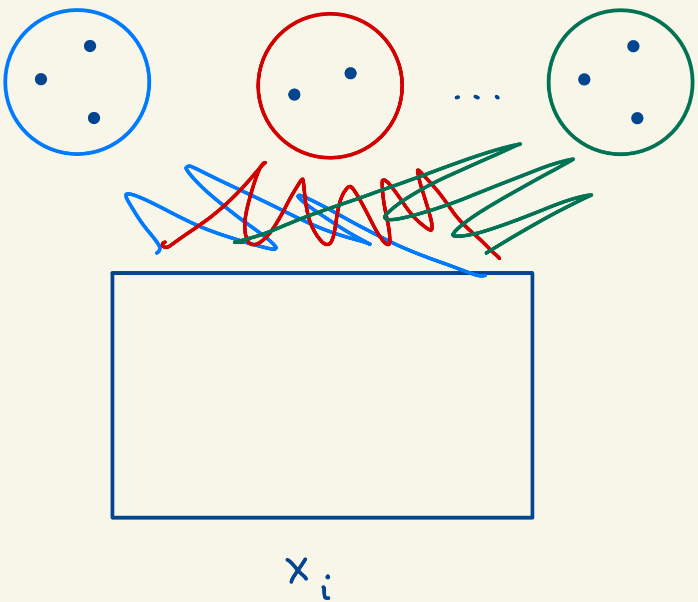

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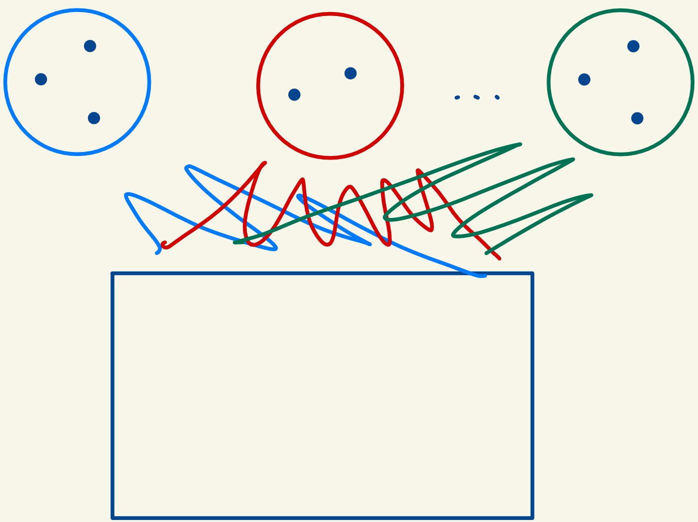

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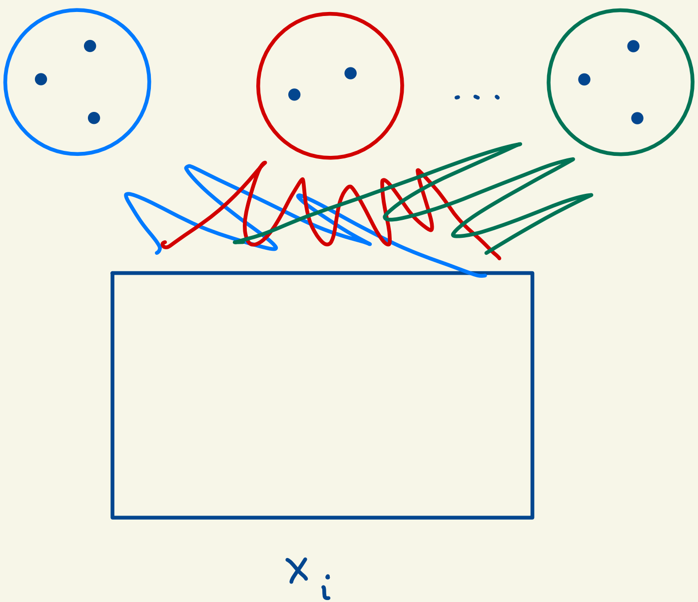

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At time $i = r(k-1)$ we have

a mono x clique! $\square$

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lower bound $\quad R_{r,r-1}(k) = 2^{\Omega(k/r)}$ $\quad$ (case $s = r-1$)

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Set colouring Ramsey bounds

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Assume $r \leq k$. $r > k$: $R_{r,r-1}(k) = \min\{n : \binom{n}{2} > r\}$

consider a random colouring (ind. and unif. in $\binom{[r]}{s}$)

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$\mathbb{P}(\text{fixed } k\text{-clique is mono}x) \leq r \left(\frac{s}{r}\right)^{\binom{k}{2}}$

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$\leq r n^k (1 - 1/r)^{\binom{k}{2}} \leq \left(n k^{1/k} e^{-k/r}\right)^k$

$n = 2^{\Omega(k/r)}$ < 1

$\square$

Set colouring Ramsey

↳ originally studied by Erdős, Hajnal and Rado (65)

↳ conjectured $R_{r,r-1}(k) \leq 2^{\delta(r)k}$ for some $\delta(r) \to 0$ as $n \to \infty$

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$$2^{\Omega(k/r)} \leq R_{r,r-1}(k) \leq r^{O(k/r)}$$

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"Simple bounds"

$$2^{\Omega(k/r)} \leq R_{r,r-1}(k) \leq \left(r/_{r-1}\right)^{rk}$$

Set colouring Ramsey

general setting :

"Simple bounds"

$$2^{\Omega(k(r-s)/r)} \leq R_{r,s}(k) \leq (r/s)^{rk}$$

Set colouring Ramsey

general setting :

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$$2^{\Omega(k(r-s)/r)} \leq R_{r,s}(k) \leq (r/s)^{rk}$$

Conlon, Fox, He, Mubayi, Suk and Verstraëte (22+)

$$R_{r,s}(k) = 2^{\Theta(kr)}$$

for every $s = s(r)$ s.t. s/r is bounded away from 0 and 1

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for every $s = s(r)$ s.t. s/r is bounded away from 0 and 1 (u.b. and l.b. exponents differ by $O(\log r)$)

↳ lower and upper bounds diverge significantly when

$s = o(r)$ or $r - s = o(r)$

Set colouring Ramsey

Conlon, Fox, He, Mubayi, Suk and Verstraëte (22+)

$$R_{r,s}(k) = 2^{\Theta(kr)}$$ → best upper bound known!

for every $s = s(r)$ s.t. s/r is bounded away from 0 and 1 (u.b. and l.b. exponents differ by $O(\log r)$)

↳ lower and upper bounds diverge significantly when

$$s = o(r) \quad \text{or} \quad r - s = o(r)$$

ex.:

$$2^{ck/\sqrt{r}} \leq R_{r,r-\sqrt{r}}(k) \leq 2^{c' k \log r}$$

Set colouring Ramsey

Conlon, Fox, He, Mubayi, Suk and Verstraëte (22+)

$$R_{r,s}(k) = 2^{\Theta(kr)}$$

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Aragão, Collares, Marciano, M., Morris

$\exists\ C, \delta > 0$ s.t. if $s \leq r - C \log r$, then

$$R_{r,s}(k) \geqslant \exp\left(\delta \varepsilon^2 r k\right)$$

$\forall\ k \geqslant (C/\varepsilon) \log r$, where $\varepsilon = (r-s)/r$.

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$$\boxed{R_{r,s}(k) = 2^{\tilde{\Theta}(\varepsilon^2 r k)} \quad \forall\ k \geq (1+\delta)/\varepsilon + 1}$$

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Set colouring Ramsey

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Remark :

	L.B.	exp. Δ to U.B.
$r - s = \Omega(r)$	CFHMSV	$O(\log r)$
$s = r - o(r)$ and $s \geq r - C\log r$	simple bound	$O((\log r)^2)$
$s \leq r - C\log r$	ACMMM	$O(\log r)$

Lower bound construction

$s \leq r - c\log r$, $n = \exp(\delta\varepsilon^2 rk)$

↳ WTS: $\exists$ a colouring $c : E(K_n) \to \binom{[r]}{s}$ with no K_k mono χ.

proof:

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For each color $i \in [r]$ consider

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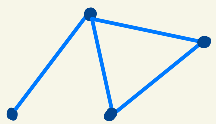

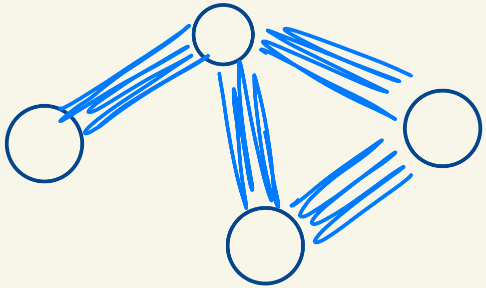

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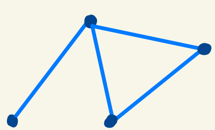

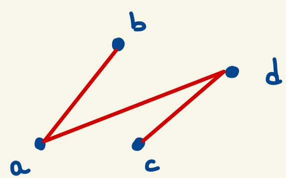

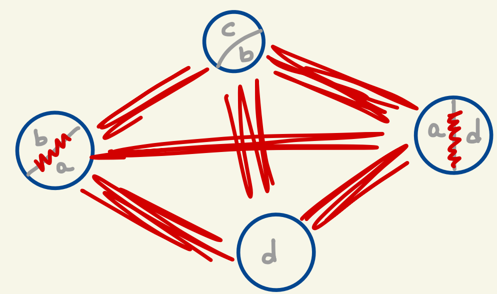

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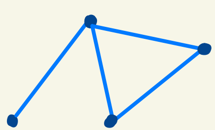

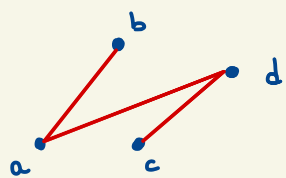

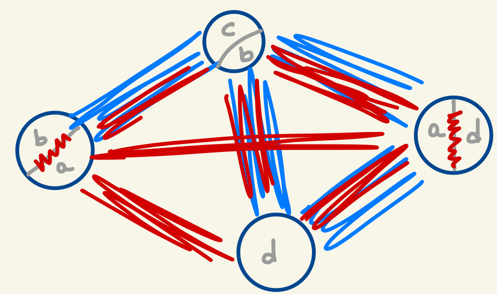

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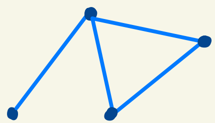

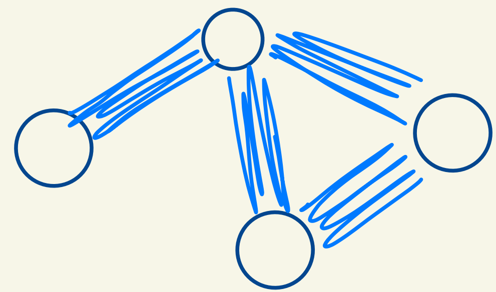

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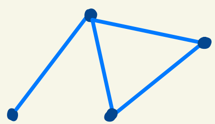

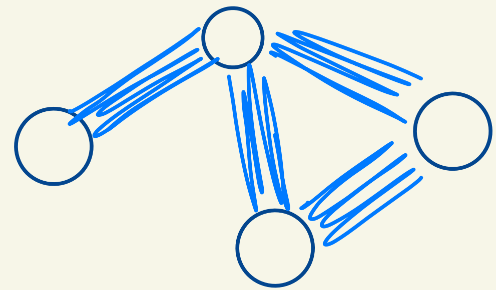

$\chi'(uv) = \{ i \in [r] : \{\phi_i(u), \phi_i(v)\} \in E(H_i) \}$ $\forall$ $uv \in E(K_n)$

Bad edges : $B = \{ e \in E(K_n) : |\chi'(e)| < s \}$

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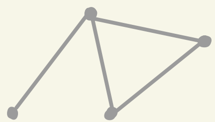

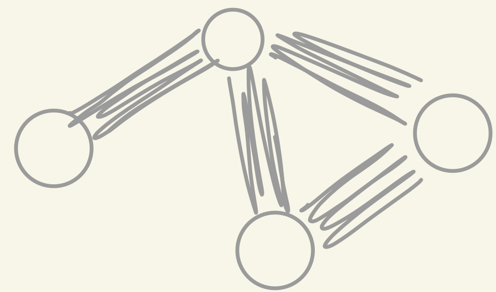

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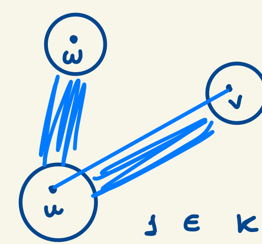
uv-crossing colours: $K(uv) = \{ i \in [r] : \phi_i(u) \neq \phi_i(v) \}$



$1 \in K(uv)$

$1 \in K(vw)$

Lower bound construction

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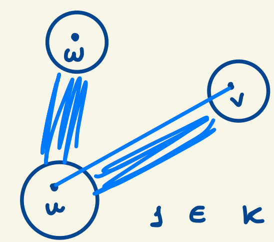

$j \in K(uv)$

$j \in K(vw)$

Lower bound construction

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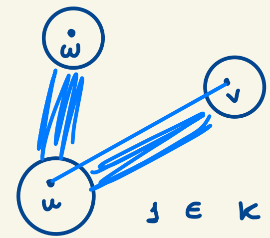

Crossing colours: $K(uv) = \{ i \in [r] : \phi_i(u) \neq \phi_i(v) \}$

$j \in K(uv)$
$j \in K(vw)$

Colouring:

$$\chi(e) = \begin{cases} \chi'(e) & \text{if } e \notin B \\ K(e) & \text{if } e \in B \end{cases}$$

Lower bound construction

Properties we want (choice of parameters)

"Almost all" edges of χ' have $\geq s$ colours $\hookrightarrow$ B is small

$\hookrightarrow$ $p \geq 1 - c\varepsilon$ (taking p small helps)

Lower bound construction

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Choose m s.t.

every k-set misses $\Omega(\varepsilon k^2)$ edges in every colour class.

$\hookrightarrow$ $G(m, p)$ is unlikely to contain a K_k

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Properties we want (choice of parameters)

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Choose m s.t.

every k-set misses $\Omega(\varepsilon k^2)$ edges in every colour class.

↳ $G(m, p)$ is unlikely to contain a K_k

"backup colouring" is s.t. no edge misses more than εr colours

↳ $m = 2^{\delta^2 \varepsilon k/2}$ satisfies both!

Lower bound construction

χ is a valid colouring!

- Every edge receives s colours

  ↳ definition + $|K(e)| \geq s \quad \forall e \in E(K_n)$ (first moment)

Lower bound construction

χ is a valid colouring!

- Every edge receives s colours

  ↳ definition + $|k(e)| \geq s \quad \forall e \in E(K_n)$ (first moment)

- Whp no K_k mono χ.

  ↳ cliques with few bad edges (Chernoff)

  ↳ cliques with many bad edges ($\geq \delta \varepsilon k^2$)

  ↳ technical part

  need to use randomness of the partitions ϕ_i's

$\square$

Open problems

- reduce the $(\log r)^2$ gap when $r - s = c \log r$ and $\varepsilon k = \log r$.

- hypergraph set colouring Ramsey number

  ↳ problems in CFHMSV:

  prove $R^{(3)}_{5,2}(k)$ is double exponential in n

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  ↳ problems in CFHMSV:

  prove $R^{(3)}_{5,2}(k)$ is double exponential in n

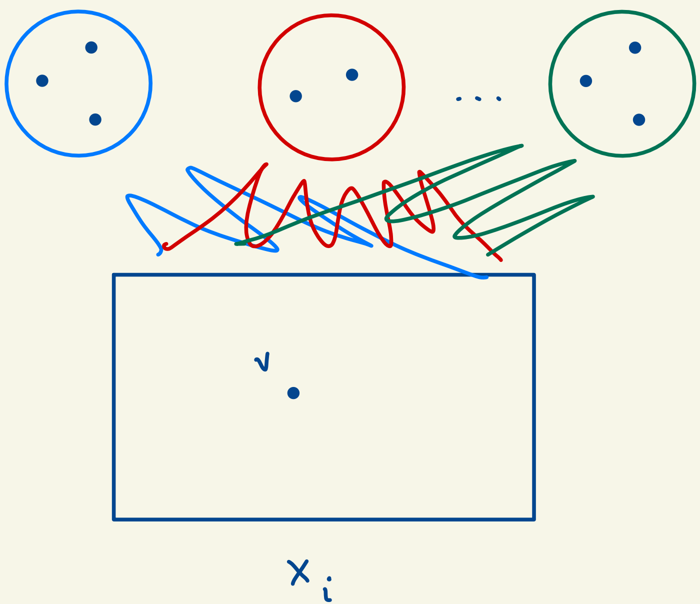
Thank you!

Set colouring Ramsey bounds

upper bound (Conlon, Fox, He, Mubayi, Suk and Verstraëte)

$$R_{r,s}(k) < 2^{\Theta(kr)}$$

sketch:



Traditional Erdős-Szekeres:

neighbourhood chasing argument

+

keeping track of size of

mono χ cliques

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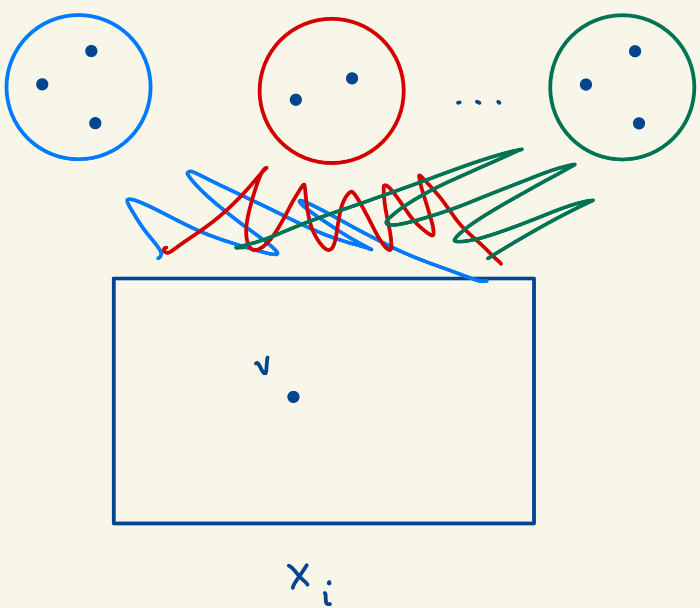

~~Traditional~~ Modified Erdős-Szekeres:

neighbourhood chasing argument
↳ only in active colours
+
keeping track of size of mono X cliques
↳ for active colours

Set colouring Ramsey bounds

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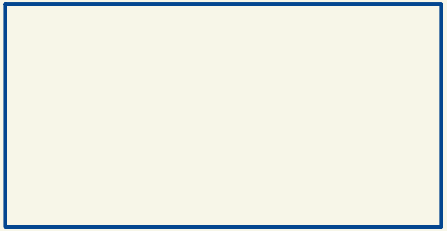
sketch:

no active colours



X_0

Set colouring Ramsey bounds

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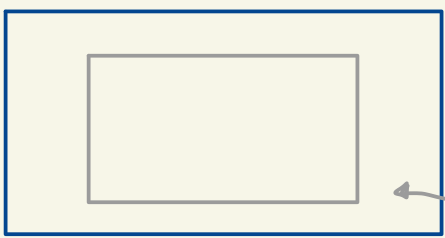
sketch:

no active colours

choose densiest colour in X_0 to be activated (blue)



X_0

reasonably dense in blue

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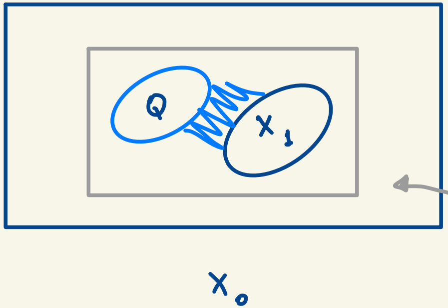

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Set colouring Ramsey bounds

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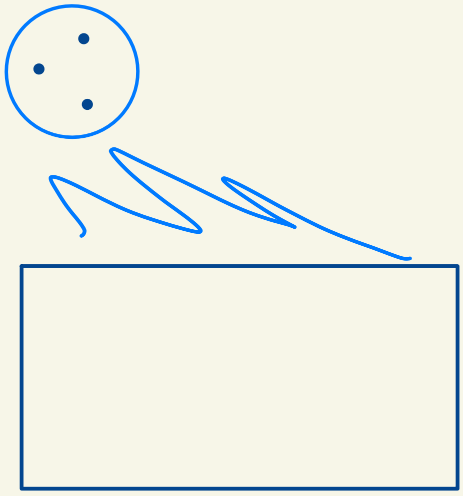

active colours

X_1

Set colouring Ramsey bounds

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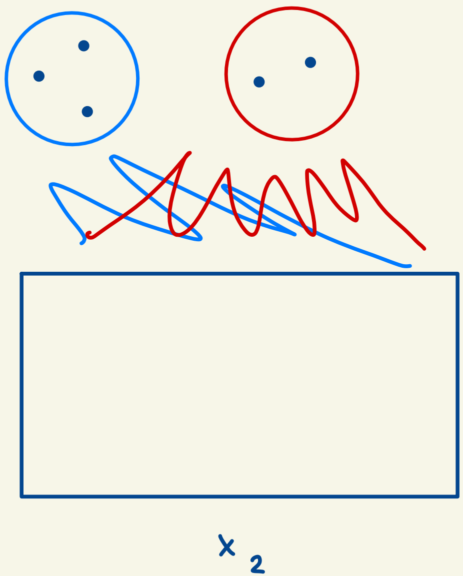

active colours

Do the densities of all active colours are bellow 1/2 in X_i ?

↳ yes, activate new colour.

Set colouring Ramsey bounds

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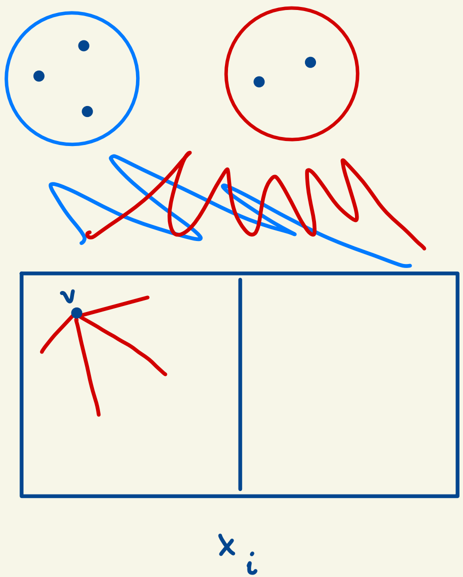

active colours

Do the densities of all active colours are bellow $1/2$ in X_i ?

↳ yes, activate new colour.

↳ No ! density of red $\geq 1/2$

$$|N_R(v)| \geq |X_i|/2 .$$

Set colouring Ramsey bounds

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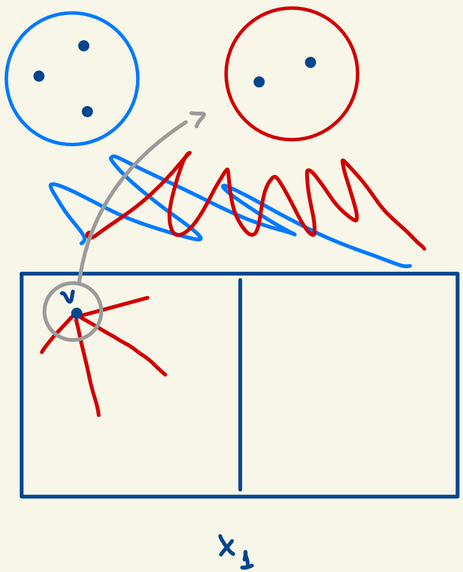

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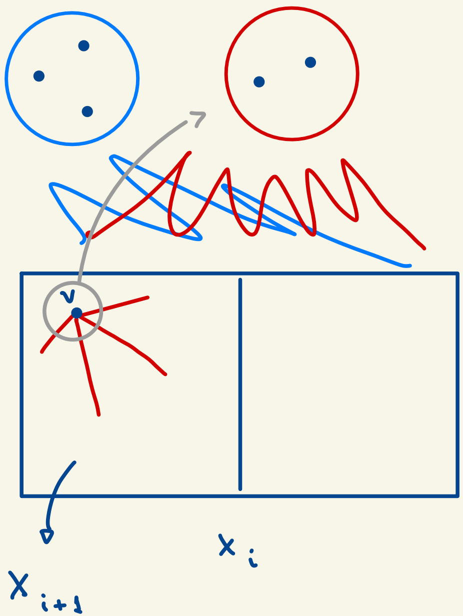

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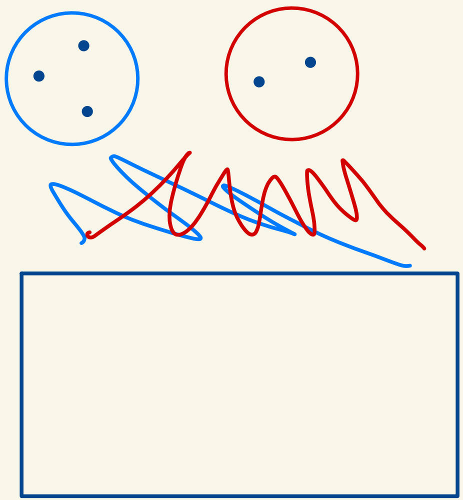

X_{i+1}

active colours

Do the densities of all active colours are bellow 1/2 in X_i?

↳ yes, activate new colour.

↳ No! density of red $\geq 1/2$

$$|N_R(v)| \geq |X_1|/2 .$$

$\square$