Probability Seminar - IM/UFRJ

Law of large numbers for ballistic random walks in dynamic random environments under lateral decoupling

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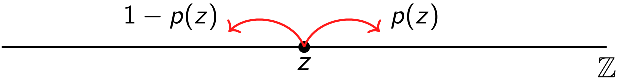

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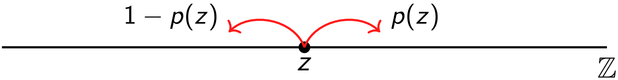

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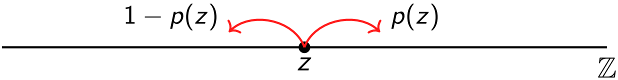

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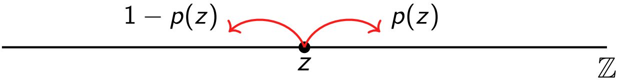

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  - We assume that η invariant by space-time translations.

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  f_1 and f_2 two non-negative functions, $||f_1||_\infty \leq 1$, $||f_2||_\infty \leq 1$.

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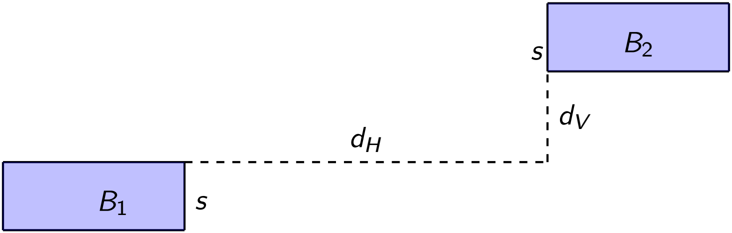

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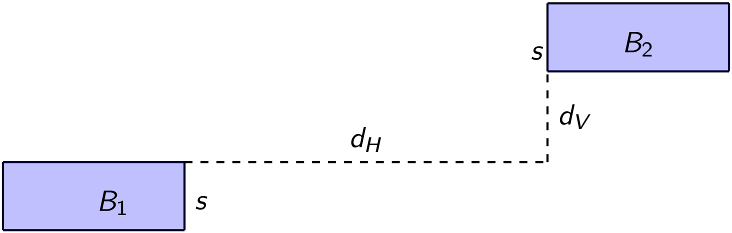

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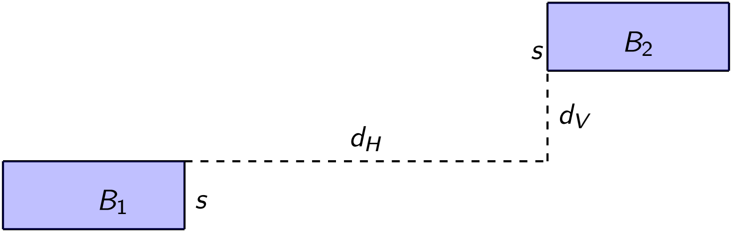

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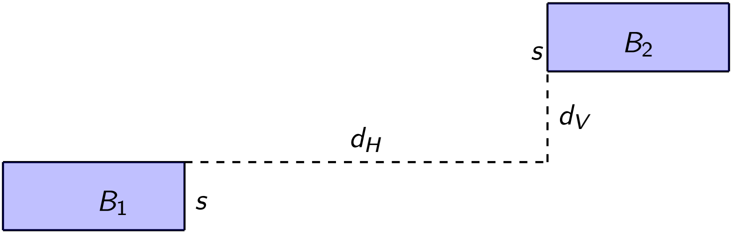

If $d_H \geq v_\circ d_V + C_1 s + C_2$, $\quad v_\circ, C_1, C_2,$ positive constants, then

$$\mathbb{E}[f_1 f_2] \leq \mathbb{E}[f_1]\mathbb{E}[f_2] + C_\circ e^{-\kappa_\circ (\log d_H)^{\gamma_\circ}}, \quad C_\circ, \kappa_\circ \in \mathbb{R}_+, \gamma_\circ > 1.$$

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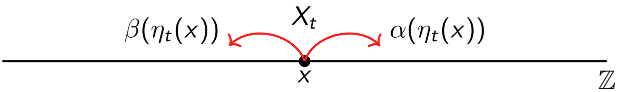

  - $\sup_{\xi \in E} \{\alpha(\xi) + \beta(\xi)\} \leq \Lambda$.

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  There exist constants $v_\star, \kappa_\star, C_\star > 0$ and $\gamma_\star > 1$ such that

$$\mathbb{P}\left(X_t \leq v_\star t\right) \leq C_\star e^{-\kappa_\star (\log t)^{\gamma_\star}} \qquad \text{for all } t \geq 0.$$

Main theorem

Theorem

Assume that Assumptions DEC and BAL hold with $v_\star > v_\circ$. *Then there exists a speed* $v \geq v_\star$ *such that*

$$\lim_{t\to\infty} \frac{X_t}{t} = v \quad \mathbb{P}\text{-almost surely.}$$

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- $H \geq 1$, $v \in \mathbb{R}$, $w \in \mathbb{R}^2$

$$A_{H,w}(v) = \Big[\exists\ y \in I_H(w) \cap \mathbb{L} \text{ s.t. } X_H^y - \pi_1(y) \geq vH\Big],$$

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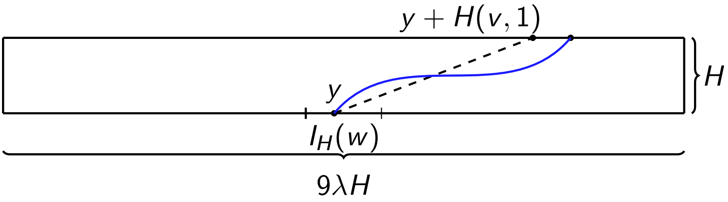

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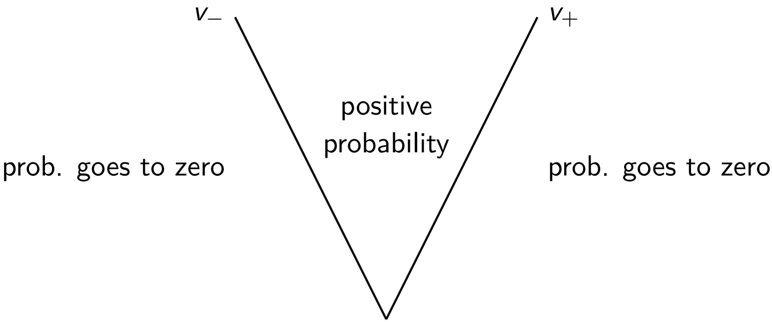

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$\forall \varepsilon > 0$, $\exists c_1 = c_1(\varepsilon) > 0$ *such that*

$$p_H(v_+ + \varepsilon) \leq c_1 e^{-3\kappa \log^\gamma H} \quad \textit{and} \quad \tilde{p}_H(v_- - \varepsilon) \leq c_1 e^{-3\kappa \log^\gamma H},$$

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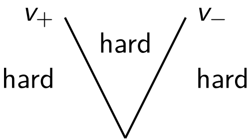

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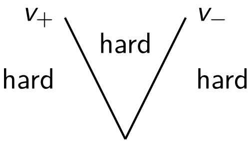

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- $m = (h, k, w)$

$$B_m := B_{L_k}^h(w), \;\; A_m(v) := A_{hL_k,w}(v) \text{ and } \tilde{A}_m(v) := \tilde{A}_{hL_k,w}(v).$$

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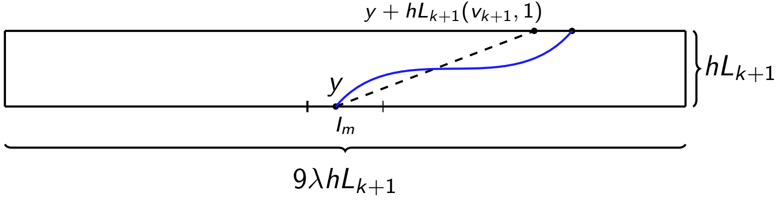

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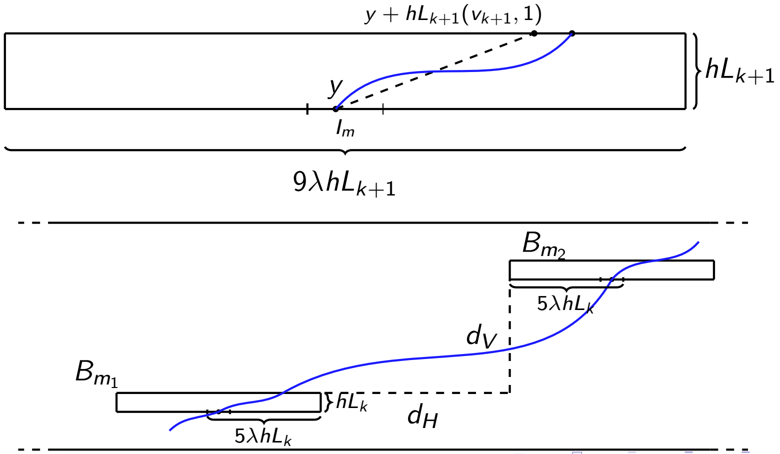

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- The other case is similar

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- $\Gamma_- \leq g(k) - g(k-1) \leq \Gamma_+$, for all $k \geq 1$.
- Marginals: for $\phi \in \mathbb{R}_+$

$$\nu_\phi(k) = \frac{1}{Z(\phi)} \frac{\phi^k}{g(k)!}, \quad \text{for all } k \in \mathbb{N}_0 \text{ and } x \in \mathbb{Z}.$$

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- Evolution:

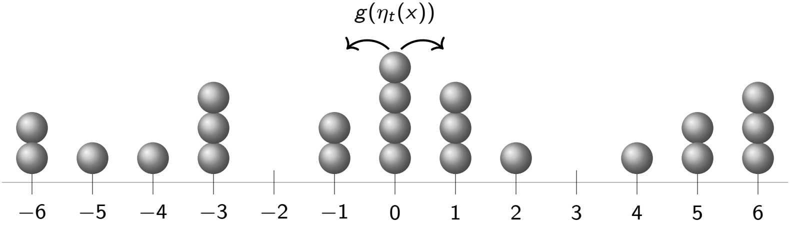

Figure: The evolution of the zero-range process.

Decoupling[R. Baldasso, A. Teixeira]

- f_1 and f_2 two non-negative functions, $||f_1||_\infty \leq 1$, $||f_2||_\infty \leq 1$.

Decoupling[R. Baldasso, A. Teixeira]

- f_1 and f_2 two non-negative functions, $||f_1||_\infty \leq 1$, $||f_2||_\infty \leq 1$.
- f_i supported on B_i.

Decoupling[R. Baldasso, A. Teixeira]

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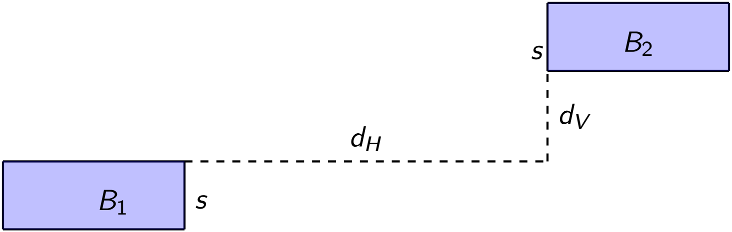

Decoupling[R. Baldasso, A. Teixeira]

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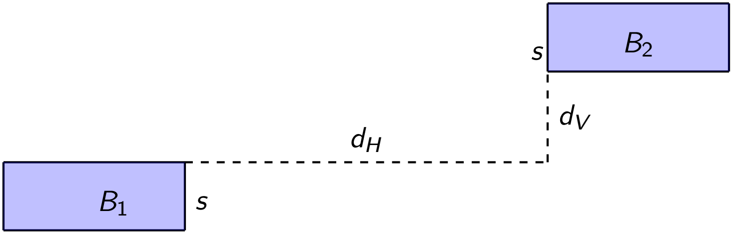

- $d_H \geq C_1(s + d_V) + C_2$.

Decoupling[R. Baldasso, A. Teixeira]

- f_1 and f_2 two non-negative functions, $||f_1||_\infty \leq 1$, $||f_2||_\infty \leq 1$.
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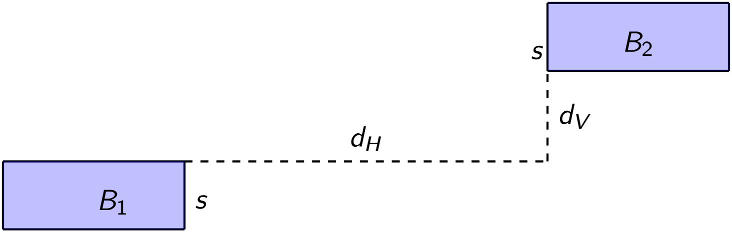

- $d_H \geq C_1(s + d_V) + C_2$.
- $\mathbb{E}^\rho[f_1 f_2] \leq \mathbb{E}^\rho[f_1]\mathbb{E}^\rho[f_2] + c_1 e^{-c_1^{-1} \log^{5/4} d_H}, \quad \gamma_\circ = 5/4.$
- Other example: **Asymmetric exclusion process**

Future works

- Law of large numbers

Future works

- Law of large numbers
- CLT

Future works

- Law of large numbers
- CLT
- higher dimensions

Future works

- Law of large numbers
- CLT
- higher dimensions

Thank You!

References

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