Probability Group - PPGE

Instituto de Matemática - UFRJ

Minicurso

Título: The cutoff phenomenon for stochastic Langevin equations

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Data: 4 a 8 de dezembro de 2023

Horário: 2a, 4a às 15:30h; 3a, 5a, 6a às 13:00h

Local: Sala C116 (2a, 3a, 5a), Sala B108-A(4a), Sala C119 (6a)
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Abstract: The cutoff phenomenon is a classical threshold phenomenon for the thermalization of a given stochastic model, such as random walks on finite groups modelling card shuffling, for instance, to its equilibrium along a certain time scale. More precisely, assume a parametrization of a family of processes, its limiting measures, and a family of renormalized distances. The mentioned parameter can be for example the size n of a deck of cards or (in the case of a stochastic differential equation) the noise amplitude epsilon. The cutoff phenomenon establishes a parameter-dependent time scale t such that in the limit of the parameter the distance between the current state is ever sharper divided between *large* values of the distance, when the system lags behind the time scale t (”to the left”) and *small* values when the system is ahead of t (”to the right”).
It has been studied in many discrete situations (time and space) since the beginning by the seminal works of Aldous and Diaconis. In most settings it is shown in terms of the total variation distance, which in continuous space presents a too strong topology in general since it is discontinuous under discrete approximations, which leads to somewhat artificial smoothing assumptions on noise and the invariant measure in several higher-dimensional settings.
In a series of papers, the speaker and co-authors have studied the cutoff phenomenon for linear and nonlinear stochastic (partial) differential equations with small and non-small noise mostly in the Wasserstein distance. This minicourse offers an introduction to the subject and its proof methods.

Lecture 1. The cutoff phenomenon for Brownian Langevin equations in one dimension for small noise in total variation. Extension to the multivariate Lévy case under sufficient regularity assumptions.
Lecture 2. The cutoff phenomenon for a dissipative nonlinear Langevin equation with small alpha stable noise.
Lecture 3. The cutoff phenomenon in Wasserstein distance without the restrictive hypotheses of the previous lectures. Extension to rather irregular systems such as linear chains of random oscillators with degenerate noise.

Lecture 4. Generalizations to a nonlinear case.
Lecture 5. Getting rid of the smallness of the noise in the stochastic differential equations. Establishment of a threshold phenomenon or ”cutoff convergence”: rescaling of the thermalization distance. Applications to the multivariate Ornstein-Uhlenbeck process and a linear energy shell model of turbulence.

References:

Aldous, D., and Diaconis, P., *Strong uniform times and finite random walks*, Adv. in Appl. Math.8, no. 1 (1987) 69-97.

Barrera, J, Bertoncini, O., and Fernández, R., *Abrupt convergence and escape behavior for birth and death chains*, J. Stat. Phys. 137, no. 4, (2009) 595-623.

Diaconis, P., *The cut-off phenomenon in finite Markov chains*, Proc. Nat. Acad. Sci. U.S.A. 93, no. 4 (1996) 1659-1664.

Levin, D, Peres, Y., and Wilmer, E., *Markov chains and mixing times*, Amer. Math. Soc. Providence, 2009.

Lecture 1:

Barrera, G., and Jara, M., *Thermalisation for small random perturbation of hyperbolic dynamical systems*, Ann. Appl. Probab. 30, no. 3 (2020) 1164-1208.

Barrera, G., and Jara, M., *Abrupt convergence of stochastic small perturbations of one dimensional dynamical systems*, J. Stat. Phys. 163, no. 1, (2016) 113-138.

Barrera, G., and Pardo, J.C., *Cut-off phenomenon for Ornstein-Uhlenbeck processes driven by Lévy processes*, Electron. J. Probab. 25, no. 15 (2020) 1-33.

Lecture 2:

Barrera, G., *Abrupt convergence for a family of Ornstein-Uhlenbeck processes*, Braz. J. Probab. Stat. 32 (2018), no. 1, 188–199.

Barrera, G., Högele, M.A., and Pardo, J.C., *The cutoff phenomenon in total variation for nonlinear Langevin systems with small layered stable noise*, Electron. J. Probab. 26, no. 119 (2021) 1-76.

Lecture 3:

Barrera, G., Högele, M.A., and Pardo, J.C.,*Cutoff thermalization for Ornstein–Uhlenbeck systems with small Lévy noise in the Wasserstein distance*, J. Stat. Phys. 184, no. 27, (2021).

Lecture 4:

Barrera, G., Högele, M.A., and Pardo, J.C., *The cutoff phenomenon in Wasserstein distance for nonlinear stable Langevin systems with small Lévy noise*, J. Dyn. Diff. Equat. (2022).

Lecture 5:

Barrera, G., Högele, M.A., Pardo, J.C., and Pavlyukevich, I., Cutoff ergodicity bounds in Wasserstein distance for a viscous energy shell model with Lévy noise. `https://arxiv.org/abs/2302.13968`

Barrera, G. Högele, M.A., *Ergodicity bounds for stable Ornstein-Uhlenbeck systems in Wasserstein distance with applications to cutoff stability.* `https://arxiv.org/abs/2306.11616`

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