Controle ótimo um passo à frente via otimização e aplicações

Optimization-based one step ahead optimal control

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Outline

- What is control & optimal control?
- What is one step ahead optimal control (OSAOC)?
- Optimal control via optimization
- Some examples
- Concluding remarks

What is control?

- Dynamical system $\dot{x} = f(x)$:object of study in mathematics
- Vector field on RHS is "God-given"
- Mathematician's job is to describe evolution of state x: equilibria, limit cycles, etc.
- Control theory studies $\dot{x} = f(x, u)$ where u is a control input
- Control engineer's job is to choose control input u so that evolution described by RHS vector field behaves in prespecified fashion.
- For example, can u be chosen so that all trajectories go to (a globally stable) equilibrium, or limit cycle?

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Linear control: discrete-time case

- Dynamical system $x(k+1) = Ax(k) + Bu(k), x \in R^n, u \in R^m, A \in R^{n \times n}, B \in R^{n \times m}$: most well-understood case, complete theory.
- In this case, without control $(u(k) = 0, \forall k)$, origin is stable if spectral radius $\rho(A) < 1$.
- What if spectral radius $\rho(A) > 1$?
- Assume that state $x(k)$ is measured and available to engineer.
- Key idea of full state feedback: choose $u(k) = Fx(k)$
- Dynamical system under feedback becomes $x(k+1) = (A + BF)x(k)$
- New question: given A, B, can F be chosen such that $\rho(A + BF) < 1$?
- Yes! Under simple algebraic condition (unstable "modes" are "controllable").

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What is optimal control?

- Dynamical system $\dot{x} = f(x, u)$: continuous-time case.
- Suppose that there is some mission objective: e.g., landing a spacecraft on the moon.
- Also suppose that there are physical constraints: energy (=fuel) has weight, thus limited
- New problem: design control u such that trajectory x fulfills mission & does not violate constraints.
- In most cases, we wish to do this optimally.
- Given an objective function (performance index, cost) $J(x, u) = \int_0^T (x(t)^T Qx(t) + u(t)^T Ru(t))dt$ (for example)
- Can $u(t)$ be chosen such that cost $J(x, u)$ is minimized?
- Yes! Complete theory: Bolza, Bliss, Pontryagin, Bellman.

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Optimal control: mathematical formulation

- Given a dynamical system $\dot{x} = f(x, u)$
- and a cost function $J(x, u)$
- minimize $J(x, u)$, by choice of $u(t)$, $t \in [0, T]$, subject to the dynamics $\dot{x} = f(x, u)$
- Control u is a function; trajectory x must satisfy an ODE with RHS dependent on the choice u.
- This is a difficult problem of variational calculus.
- Even when there are no additional constraints on the state x and the control u!
- Without additional constraints: complete theory in LQ case (linear system, quadratic cost)
- LQ case: optimal control is found in feedback form (solving Riccati equation or by dynamic programming).
- Kalman, Bellman: early 1960s.

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Optimal control: discrete-time case

- Given a dynamical system $x(k+1) = f(x(k), u(k)), k = 1, 2, \dots, K$
- and a cost function $J(x(k), u(k))$
- minimize $J(x(k), u(k))$, by choice of $u(k), k \in [1, K]$, subject to the dynamics $x(k+1) = f(x(k), u(k))$
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- We can now reinterpret the optimal control problem as an optimization problem.

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Optimal control as mathematical optimization

- minimize $J(x(k), u(k))$, by choice of $u(k), k \in [1, K], x(k), k \in [1, K+1]$
- subject to the K equality constraints $x(k+1) = f(x(k), u(k)), k = 1, 2, \ldots, K$
- This is a standard nonlinear programming problem, convex if J is convex and f is affine
- Easy to solve if convex, even if J is nonlinear
- **Convex constraints on x and u can be added without increasing the difficulty of solution!**
- Price to be paid: $2K + 1$ decision variables.

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- subject to the K equality constraints $x(k+1) = f(x(k), u(k)), k = 1, 2, \dots, K$
- This is a standard nonlinear programming problem, convex if J is convex and f is affine
- Easy to solve if convex, even if J is nonlinear
- **Convex constraints on x and u can be added without increasing the difficulty of solution!**
- Price to be paid: $2K + 1$ decision variables.

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Optimal control as mathematical optimization (history)

- Krasovskii (1957): Impulse response → linear program formulation (not explicitly identified as such).
- Zadeh (1962): made Krasovskii's contribution explicit as linear program.
- **Cannon, Cullum and Polak (1970): Far-reaching theoretical extensions of Krasovskii/Zadeh ideas.**
- Boyd, Barratt (1991): Linear Controller Design: Limits of Performance (convex formulations).
- Boyd et al. LMIs in System & Control (1994) (convex optimization).
- Mayne, Rawlings et al. (2000) Constrained model predictive control: Stability and optimality.
- Goodwin et al. (2005): Constrained Control and Estimation - An Optimisation Approach.
- Borrelli, Bemporad, Morari (2017): Predictive control for Linear and Hybrid Systems.

This talk

- Optimization-based one step ahead optimal approach, discrete-time, implementable in real time.
- Not (necessarily) linear quadratic (LQ).
- Examples from Business Dynamics: index which takes both the management and financial aspects into account.
- Examples from numerical algorithm design, opinion dynamics & dynamic games.
- Contributions: No prediction required, close to optimal, effect of delays, Julia+JuMP open source code.
- Drawback: Approach is deterministic, although some statistical simulation is done
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Omniscient optimal control

- Dynamical system:
  $x(k+1) = F(x(k), u(k), e(k), k), k = 1, \ldots, K_f$
- $x(k)$: state variable.
- $u(k)$: control input chosen by the decision maker, usually subject to some constraints.
- $e(k)$: **exogenous input known at time k, but not known for future times, and not under the control of the decision maker.**
- **Omniscient optimal control problem:** Given the exogenous input sequence $\{e(k)\}_{k=1}^{K_f}$ and initial state $x(1)$
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$$\begin{array}{ll} \textit{maximize} & J_{om}(K_f+1) \\ \textit{subject to} & x(k+1) = F(x(k), u(k), e(k), k) \\ \textit{by choice of} & u(k), k = 1, \ldots, K_f \end{array}$$

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- **One step ahead optimal control problem:** Given the current exogenous input $e(k)$and the current state $x(k)$
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Types of performance index/objective function & constraints

- Linear: linear combination of (nonnegative) states and controls.
- Piecewise linear: weighted combination of one- or infinity-norms of states and controls
- Quadratic: sum of quadratic forms in "errors" and controls
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Debt amortization: problem statement

- Amortizing given initial debts with target date (planning horizon) to pay off.
- Upper bounds on withdrawals from investment accounts to make amortization payments.
- Additional debts incurred during planning horizon.
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Debt amortization: mathematical model

- $$x_i(k+1) = (1+\alpha_i)x_i(k) - \sum_{j=1}^{n} u_{ij}(k) + c_i(k), \forall i$$
  $$y_j(k+1) = (1+\beta_j)y_j(k) - \sum_{i=1}^{m} u_{ij}(k) - \gamma_j \sum_{i=1}^{m} u_{ij}(k) + d_j(k), \forall j$$
- x_i:ith debt, y_j:j th investment account balance.
- α_i, β_j: ith debt and j th investment account interest rates, respectively.
- γ_j:transaction cost per unit of cash transferred from investment account.
- Box constraints on x_i,y_j, upper bounds on $u_i(k) := \sum_{j=1}^{n} u_{ij}(k)$ for each i.

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Debt amortization: Performance indices

- Weighted debt amortization with wealth maximization
- $J_{os}(k+1) = \sum_{j=1}^{n} \kappa_j y_j(k+1) - \sum_{i=1}^{m} \mu_i x_i(k+1)$
- $J_{om} = \sum_{j=1}^{n} \kappa_j y_j(K_f+1) - \sum_{i=1}^{m} \mu_i x_i(K_f+1)$

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Debt amortization+wealth maximization: LP formulation

- maximize J_{om}(resp.J_{os}) [linear objective function]
- subj. to dynamics for all k(resp. at k) [linear equality constraints]
- and subj. to box and upper bound constraints for all k (resp. at k) [linear inequality constraints].

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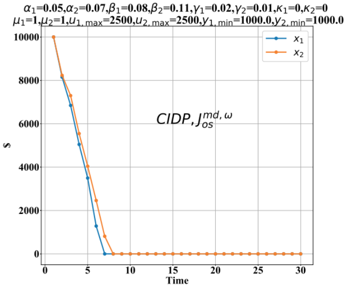
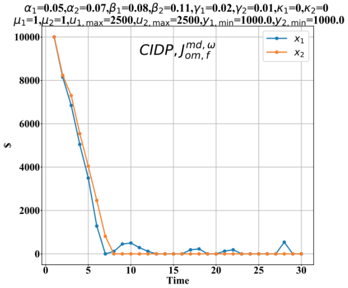
Debt amortization+wealth maximization: example

	Mean	Standard deviation	Property
Expenditures c_1	565	311	Random
Expenditures c_2	501	288	Random
Deposits d_1	345	147	Random
Deposits d_2	320	231	Random

Example: evolution of debt sequences



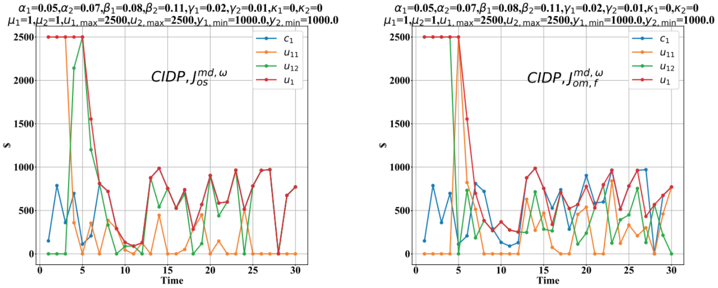
(a) Evolution of debts 1 and 2 under one step ahead policy, maximizing $J_{os}^{md,\omega}$.



(b) Evolution of debts 1 and 2: omniscient solution, maximizing $J_{om,f}^{md,\omega}$.

Debt sequences 1 and 2 for OSA and omniscient optimal controls are similar, but not identical.

Example: amortization payments

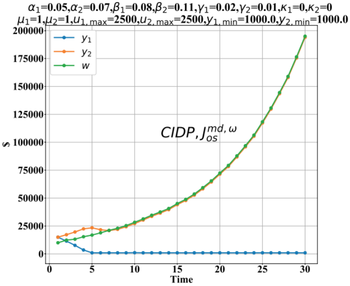


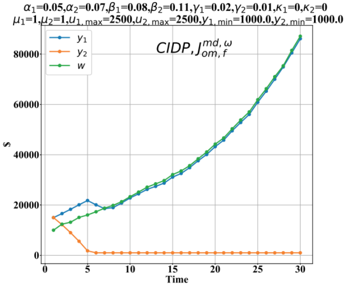
(a) One step ahead amortization payments for debt 1, maximizing $J_{os}^{md,\omega}$.

(b) Omniscient amortization payments for debt 1, maximizing $J_{om,f}^{md,\omega}$.

Total amortization payment $u_1 = u_{11} + u_{12}$ tracks expenditure c_1, after initial period at $u_{1,max}$.

Example: investment account balances





OSA: Investment account 1 $\rightarrow$minimum. Omniscient: Investment account 2 $\rightarrow$minimum.

Debt amortization: choice of weights

Row	Index	κ_1	κ_2	μ_1	μ_2	Final debt	Final wealth	Comments
1	$J^{md,\omega}_{om,f}$	0	0	1	1	0	93011	$x_1, x_2 \downarrow 0, y_2 \downarrow y_{2,\min}$
2	$J^{md,\omega}_{os}$	0	0	1	1	**0**	**215305**	$x_1, x_2 \downarrow 0, y_1 \downarrow y_{1,\min}$
3	$J^{md,\omega}_{om,f}$	1	1	1	1	22703	322604	$y_1 \downarrow y_{1,\min}$
4	$J^{md,\omega}_{os}$	1	1	1	1	20000	160026	$y_1 \downarrow y_{1,\min}$
5	$J^{md,\omega}_{om,f}$	1	5	10	10	0	319202	$x_1, x_2 \downarrow 0, y_1 \downarrow y_{1,\min}$
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8	$J^{md,\omega}_{os}$	1	1	10	10	0	92731	$x_1, x_2 \downarrow 0, y_2 \downarrow y_{2,\min}$

- Similar outcomes: Row 2 OS (implementable)/Row 7 Omni (unimplementable) = 67%.
- **Overall conclusion: choice of weights $\kappa_i = 0, \mu_j = 1, \forall i, j$ (debt amortization) is good candidate for real time debt management.**

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Cash balance problem: description

- Every firm/financial institution has cash current account to meet daily transaction requirements.
- Cash balance: large enough for requirements, not much larger.
- Extra cash earns larger returns in an investment account.
- **Cash flows**, both deposits and withdrawals, into *current accounts with low interest rates.*
- **Transfers from investment accounts** to make negative current account balances positive.
- Transfer excess balance in current accounts to investment accounts (better returns).
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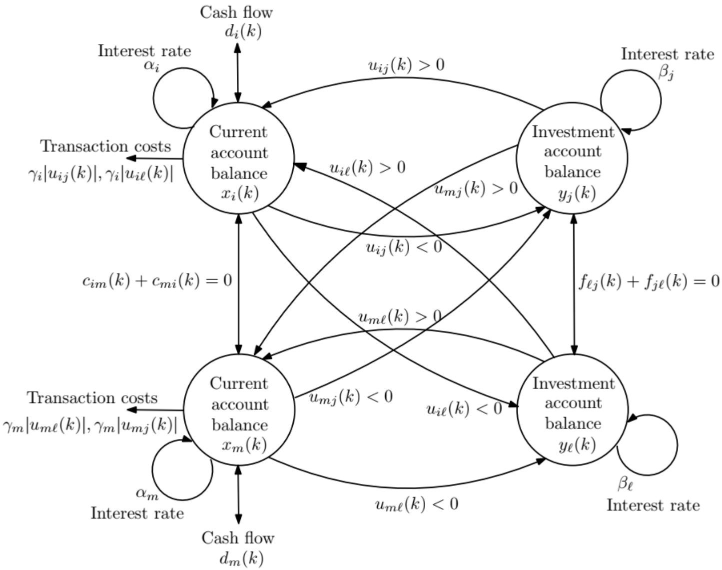
Cash balance: notation



Cash balance dynamics

$$
\begin{aligned}
x_i^+ &= (1+\alpha_i)x_i + \sum_{j=1}^{M} u_{ij} - \sum_{j=1}^{M} \gamma_i |u_{ij}| - d_i + \sum_{m \neq i} c_{im}, \forall i, \\
y_j^+ &= (1+\beta_j)y_j - \sum_{i=1}^{N} u_{ij}, \forall j, \\
0 &= c_{im}(k) + c_{mi}(k), \forall i, \forall m, \\
0 &< x_{i,\min} \leq x_i(k) \leq x_{i,\max}, \forall i, \\
0 &< y_{j,\min} \leq y_j(k), \forall j.
\end{aligned}
$$

Total wealth: $w(k) = \sum_{i=1}^{N} x_i(k) + \sum_{j=1}^{M} y_j(k)$

Cash balance problem: performance indices

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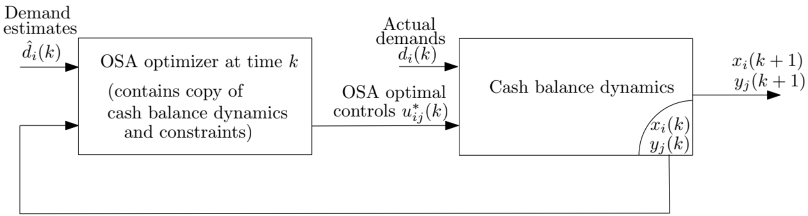
Cash balance: OSAOC block diagram



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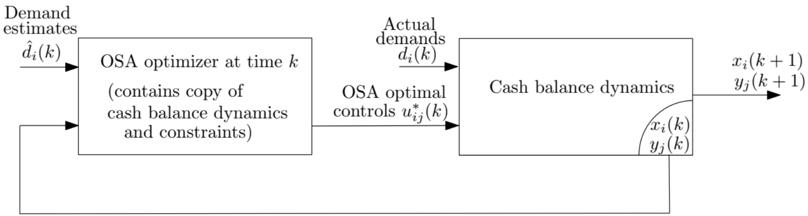

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Cash balance: classical Miller-Orr scheme (1966)

- Feedback control with dead zone, no explicit use of demand.
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Cash balance: average performance of OSAOC

- Comparison of average and relative wealth gain (AWG,RWG).
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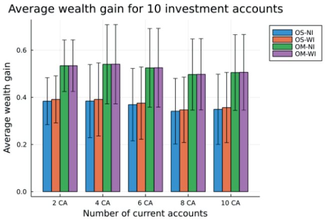
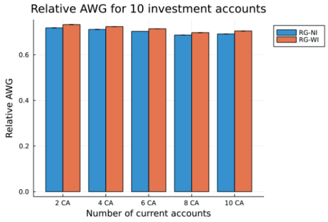
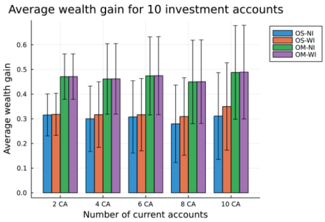


(a) Average wealth gain



(b) Relative average wealth gain



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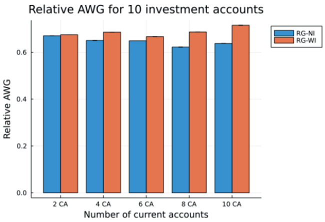
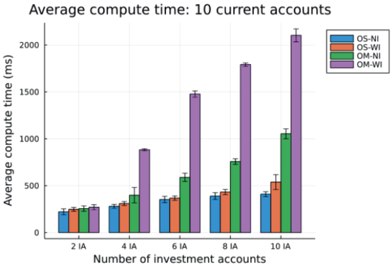
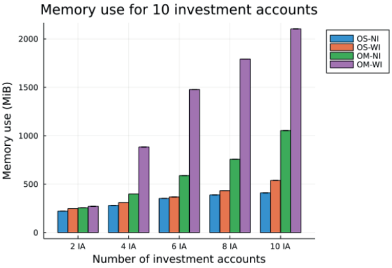


(b) Relative average wealth gain

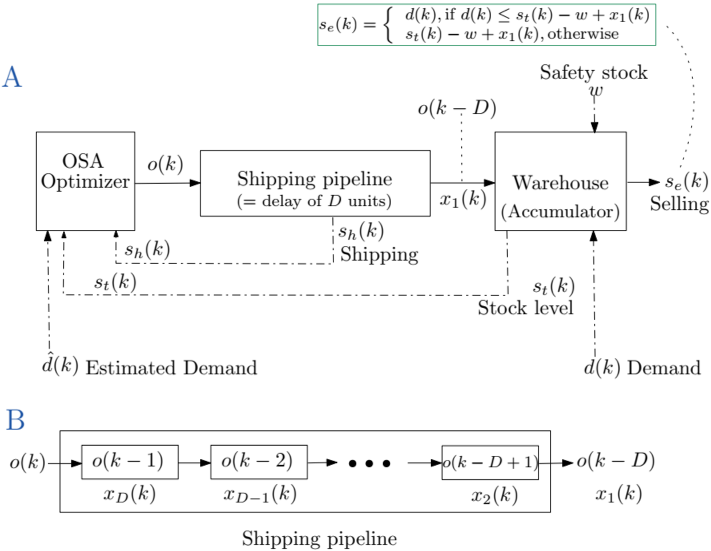
Cash balance scalability: OSA vs. Omni





- Hierarchy OSA_NI $\leq$OSA_WI $\leq$OM_NI $\leq$OM_WI
- Variance of AWG decreases as $N_{CA}, N_{IA} \uparrow$ in both cases (current demand known/unknown).
- AWG $\approx$ 40%, RWG $\approx$ 70% (normal demand), AWG $\approx$ 30%, RWG $\approx$ 60% (uniform,Bernoulli)
- Compute time, memory requirements: scale linearly (OSAOC), scale exponentially (Omni)

Inventory control and management



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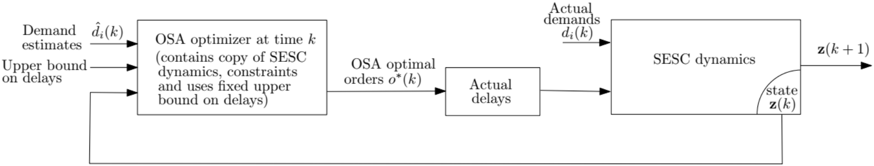
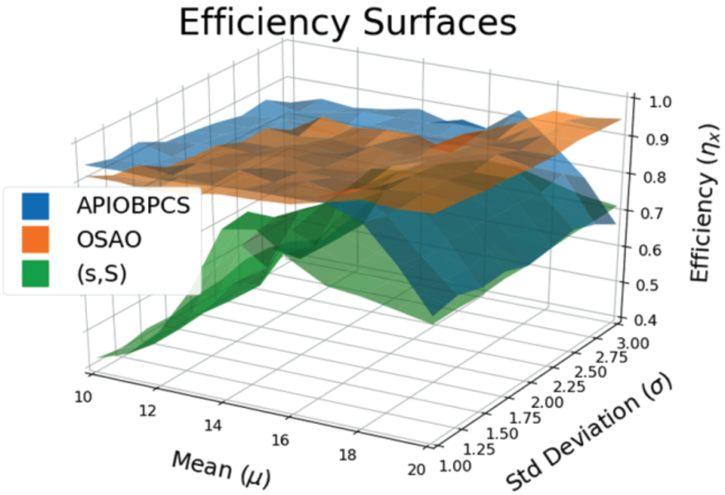


Inventory control: robustness to demand uncertainty



Inventory control: robustness to lead time uncertainty

Controller	Variable delay efficiency	Constant delay efficiency
APIOBPCS	63.65 ($\pm$ 8.38)%	83.44%
OSAO	70.32 ($\pm$ 3.68)%	85.69%
(s, S)	48.44 ($\pm$ 8.96)%	66.06%

OSAO inventory control: summary

- Novel scheme: no prediction, robust to demand, delay uncertainties.
- Uses Economic Value Added as objective function.
- Efficient, attaining 80% of (unattainable) omniscient global optimum.
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ODG+OSAOC: main results

- Closed-form expression for OSAOC.
- OSAOC results in computable Nash equilibria under J, GS and RGS game-playing procedures.
- For each player, OSAOC control = approximate projection of "residual error" onto "control direction".
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ODG under OSAOC: flipping polarization

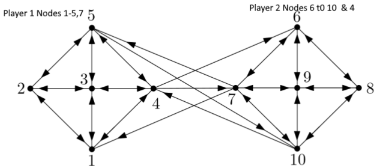
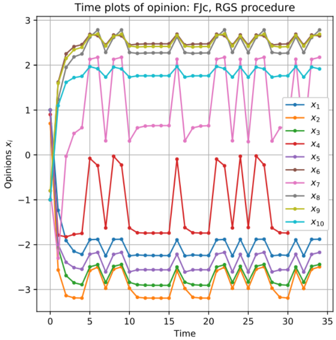


Figure 11. Directed graph of ten polarized agents in 2 clusters: agents $\{1,2,3,4,5\}$ have initial opinions close to 1 and agents $\{6,7,8,9,10\}$ have initial opinions close to -1. Two players target these clusters with the objective of flipping the polarization.



Concluding remarks

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